ON THE KNOWABILITY OF THE WORLD: FROM INTUITION TO TURING MACHINES AND TOPOS THEORY

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ABSTRACT. *Is an entire (Biocosmological) image of animate and inanimate nature can be built on the notion of discrete quantities-objects and their relations – in essence? Just as its thoughtful reflection in consciousness (as knowability of what exists), which would mimic some (sufficiently complex) mechanical device, in the manner of "artificial intelligence"? We have so that the notion of Turing machine, derived on this basis, has been "extended" to an abundance of (convincing) arguments in our time that it can truly be achieved to a high and highest extent, so it seems that the term "program" has grasped with more happiness our intuition, than is the case, say, with the concept of natural number at all.*

We present here summarily the ontological points of view – from Thales to Hegel – on the being, from an intuitive picture on it as "one and many", to the set-theoretical one, where parts-components enter the wholes, realizing mutual relations in it, while preserving their originality, and which is approached to a high degree just the mathematical theory of topoi (of category). In favor of is the position of Alain Badiou: "Mathematics is ontology and so on, so this theory can be seen as an exact (mathematical) expression of the general principles set by biocosmology: the universality of life-processes (organicism), three-dimensionality (of sphere of activity) and four-causality of causes (in the sense of Aristotle).

On the epistemological level, it would be an image of knowability of world "for us" – if not "per se" – achieved in ***concepts****, and, as (only) more or less reliable through programs, models. From that place it should be foreseen "the fundamental (universal) laws" in the nature, such as laws of evolution, etc., as well as the human ability, before the temptations of the elements of the world (environmental, demographic, etc.), to find favorable outcomes for himself.*

KEYWORDS**:** *Biocosmology, notion, Turing machine, program, topos theory*

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Contents

Introduction: "The world is one". On its knowability

In the Bible we see that the creation of the world has been achieved in such a way that God made (*ex nihilo*) the heaven and the earth, after the idea he carried in himself and then – "for he saw that it was good" [Genesis 1:31] – he settled "the earth waste and without form" [ibid., 1:2]. One after the other, with grass, plants, animals ... and finally, the sixth day, with man, to "rule over the fish of the sea and over the birds of the air" [ibid, 1:28]. He gave everything a name and a place in the world, for he made it as an order, as wholeness, embodied in his divine word, but when Adam and Eve disobeyed it as such, it resulted in the cruelest punishment for humankind: in death, in loss of eternal life, in damnation thrown forever. Because man offended here the *whole* of the being, which, though uncreated as this stone, this plant, this star ... has acquired a value "per se", by the elements making it, what in its own way has expressed, say, Greek mythology too, by translating through Eros the Earth (Gea) from *chaos* to *cosmos*, etc.

Because, it is said, "a whole is something more than parts making it", or even if without parts it cannot be imagined, it is not however reducible to them. So to rethink the double character of the whole and especially, to go behind its definition, it would be of a primary philosophical importance. Here of course it would be about the relationship between a particular and a general – or, better, something the most general what is the world – that entails the way itself of existence of the general, or the (im)possibility this notion to be subject to changes, etc. Just as in the case of the table on which I write and the notion of table (in general), for a table is not, for example, only trestle taken away, none a rectangular area on the side of it, but that – as something more – which arises from the relation of these parts. If we have to determine the whole of the world too, it would appear a difficulty of the same nature as in the definition, say, of the (general) notion of table. In the following sense: which

marks can include it as essential ones – with all porosity of the very notion of "essential" – or has the world its own purpose (mission, role), as the table, the house, the plant have it? These are the Socratic questions too about the possibility of knowledge as general and necessary one, from the very beginning of philosophical reflection of the world, while the development of science throughout history until today gave the right to (the same Socrates) belief in knowledge in the way of notion. Can we therefore bring closer the whole of the universe, at least schematically, to a definition of it, (possibly) on its way from inorganic to organic nature, or to an increasing complexity of it, or to tendency to produce life, mind, and consciousness?

On the epistemological level, it is a question of whether our cognitive powers: senses, intellect, mind… are capable to go behind the structure of the world being and "to come across the trail" of those permanent changes in it from the beginning? Or at least in some "large" parts of it, such as nature, society, history, ... And, in particular, are human powers such that can "communicate" to these parts (even) goals to which they could (or should) tend, and to direct events towards a predictable outcome? Insofar as the natural powers of man are enlarged, in our time, for unimaginable possibilities of information technologies (hardware and software), which can mimic to a high extent the operations performed by human mind in reasoning and deciding.

Let us forward somewhat how the idea of the whole of the being – in its ontological and epistemological aspect – has been "refracted" in the mind of (some) prominent philosophers from Antiquity to the present, and how it became possible, more clear, and more distinct and so flexible for an artificial (mechanical) approach, in the way of computer programs today.

1. Presocratics

First, in Presocratics, we have that the beginning of philosophy as a science (happily) began in the sign of this idea, when Thales of Miletus has said about the whole of being: "The world is one" [Diels A 13 b], although with a little luck to name it properly and to explain it. In fact, what he has seen through in his mind was that the world being, at all places, is made by one and the same, what he named "water", without going into the very structure of it. Heraclitus pointed afterwards to a new label of it: a continuous changeability of being without end, embodied here in the blink of flame of fire, governing according to the laws of *logos*, or *mind*, or *providence*. What happens in the world is lawful and necessary, and changes in it are inevitable, so that all events take place as "planned", to be then sensed, anticipated. Otherwise, the image of the world after him is that it is a scene of struggle of opposites, achieving in the result unity and harmony, so that it is rather a "fluid" image of "fire alive forever", of a river "in which one cannot enter twice" and the like.

We meet in Empedocles afterwards a "more stable" picture of the world as a "ball everywhere equal to itself" [ibid, B 29], permanently "based in a hard hideout of harmony" [ibid, B 27] – as God Spheros himself is too of a "round figure" here, happy about "circular solitude" [ibid, B 29] of himself. Such solid "labels" of it do

help certainly a discourse about being, making possible definability of what is about. Which is not the case, for example, with the being of Anaxagoras, appearing to him as *apeiron*, or "something without end" [ibid, B 1], as immense, immeasurable, unlimited and others. Parmenides, whose slogan "one and all" philosophically was more clear than "the world is one" of Thales, expressed himself also more specifically – say, from Empedocles – in the case of being, and differently from Heraclitus, by saying that it is full, motionless, invariably, not originated, one and only one. The definition that soon was followed by Plato too, and in the recent history has been cited by Hegel, Heidegger,...

And when it is about the knowability of the whole of the being, we have that by following the ancient principle "similar is reached by similar", in accordance with their ontological insight into the nature of the universe, Heraclitus, Empedocles and Democritus, ... explain possibility of a complete cognition of what surrounds us. In Heraclitus it is so that an individual (human) *soul-fire* takes part in the cosmic *fire-Logos* and in that pervasive flux of it. While in the case of Empedocles the point is about special *outflows* (*aporrai*), which as particles of earth, water, fire and air, separated from the body surface, pass through senses and connect with similar particles in the soul. And in Democritus too, what cognizes – the soul, and what is cognized – the external world, show the same, atomic structure, and here too we meet pieces which from the body surface penetrate the senses and reach the soul, where they form a sensation and so on.

But already with Epicurus, a comprehensive picture of "reflection" of the outside world and by the end in our mind becomes dramatically distorted in favor of the so-called "spontaneous" deviation of atoms from the way, "regardless in what time and in what place". Interestingly, almost the same words we use today when we talk about the position and the velocity of particles in quantum physics.

2. Plato, Aristotle,…

Plato, later, the understanding of Parmenides of being as a whole conveyed essentially to every single thing in the world, insofar as he brought it into connection with a timeless, disembodied and indestructible essence, which he called the "idea". That essence is away (separated) from thing, which is then its shadow, or copy, and itself its model, or pattern (paradigm). Everything that exists and does it is based on the idea and in this its cosmological function lies. Likewise, inasmuch as an idea exists independently, it has an ontological function too. Besides, a thing acquires its essence, unity, and identity after the idea, as something invariable and unchangeable in it, and what belongs to the notion of thing, to its meaning, making possible the knowing about it too. Plato, namely, was inclined here to a thesis of Eleatics that it is true only what is identical to itself, which does not come to be and does not pass away, does not change and does not share. By that the knowledge of thing is possible *a priori*, necessary and general one and in that is a logical function of an idea, which it exercises over thing.

So every idea is one and only one, but in *Parmenides* "the one" appears to Plato – paradoxically – as "many" too, for both times everything that is multitudinous, as sensual and changeable without end, can be thought only through some of its "standard", which is determined and one. And it, in turn, becomes identical and equal to itself only through common characteristics visible in the multitude. This double (dialectical) character of "one" and "many" is a logical and philosophical achievement owed to Eleatics and Plato, which got through history only this or that aspect, by this or that philosopher and scientist, and the matter is in fact to look through macrocosm – reliable enough – from microcosm, from a particular – a general one, from a part – a whole. Even if it is only about the confirmation of a principle, beyond any particular way, such a knowledge to be really achieved in a completed form. So the whole "escapes", here too, to the set of parts comprising it, bringing always something new in the result of relationships between them. This would refer following entities, distinguished by Plato, both along the "vertical" and "horizontal" line, in the first case, in the sense that if *pictures*, *living beings*, *mathematical objects* and *ideas* are four types of possible entities from its ontological sphere, it would be, in fact, a relationship between parts and the whole, as follows – this time after degree of their generality, as more or less real existence they deserve. And, in the second case as well, in any of above mentioned entities, we could recognize identical structure of a "mosaic" – this time too after degree of reality of corresponding elements.

It would be "transferred" equally to the epistemological realm, when four levels of intelligibility (as clarity, as reliability), in relation to: *reason*, *understanding*, *belief*, and *opinion*, related to ideas, mathematical objects, living beings and images, as follows. In that way, *the rational knowledge*, and it refers to ideas, is to the highest extent reliable, and the level below of it is *knowledge of understanding*, whose subject are mathematical objects. These are forms of knowledge related to the *intelligible world*, whereas to the *sensible world*, made by living beings and images, corresponds *opinion*, with its degrees *belief* and *opinion*, as follows. Otherwise, a sharp abyss dividing two worlds, separates equally two types of knowledge too, so that mathematical objects and rational knowledge appear here as a "bridge" between them. And in view of mutual relations of elements in the epistemological sphere, here is too about the relationship of parts to the whole, etc., so that Plato assigns to geometric (mathematical) objects to "introduce" into ideas, into higher level of being, into whole. After the example of a circle, say, which as a line without thickness, does not belong to the world of sensible things, being however reflectively "visible" for us, and so on.

The relationship of parts to whole, simple to complex, "lower" to "higher" – when it is about knowledge – the philosophy of Aristotle brings in more dispersed form, for we recognize here a spiral path of ascending from *sensual knowledge*, as the simplest, to the *rational* knowledge, as the most complex. Namely, from the relation of form (property) of an object and the senses occurs *perception*, after which comes the activity of *common sense*, which manifests analytical et al. power upon the sense

data. In that way emerge *imagination* and *rational imagination*, as a possible way of creation of an increased number of (sensual) images, to produce a *rational representation*, which extends further to the so-called *"hypolepsis"* (true assumption). From its side now a hypolepsis reveals three stages in it: *opinion*, *science* and *prudence*, so that all the series completes in *intellection* (*noein*) as "a common act of reason and what is reasonable". Namely, the so-called "passive reason" (*ho nus pathetikos*) in us, comprising (only) as potentialities in itself all what is intelligible in being, surrenders to the activity of another, "productive reason" (*ho nus poietikos*), which has the power to convert potentialities into the state of an actual rethinking of the world. Otherwise, finds Aristotle, the productive reason itself acquires the divine prerogative, entering from outside the human embryo at conception. Thus we see that an expressive finality characterizes his thought at this (important) point, which is the relation of man to the world, as a relation of knowledge.

In addition, the characteristic of this knowledge is, here too, that it is made (only) by parts which, of course, exist autonomously, but achieve also a supreme unity, the unity of the whole. However, there is extremely little indications, in Aristotle, how it exactly comes to it, or what is it that simple parts "give" of themselves, while creating what is complex and new one.

Similarly, in stoicism, we meet *the creative fire*, which as the finest physicality permeates all materiality, bringing components to a perfect order, by force of necessity. It equally acquires complex attributes, and one finds that the names as *reason* (*logos*), the *mind* (*nous*), *spirit* (*pneuma*) and the like belong to it.

Then, in Neo-Platonism also, the teaching of Plotinus is, above all, a teaching about the relation between the whole and the parts. There the whole being "disintegrates" into three (subordinated) parts of it, three *hypostases*, of which one higher – from its abundance – reveals of itself (emanates, radiates) the lower one. These are *One*, *Mind*, and *Soul*, where the former one resides in it, as in pure atemporality and absolute vagueness, beyond any particular being and essence. Behind One comes Nous, or Mind, which is translated as *spirit*, *thought*, because, in a virtual form, it contains all things and beings that rethinks. In the third case, it is about the Soul, which on its lowest level, nature, creates the visible world, after the patterns of the intelligible world (Mind), and so on. However, Plotinus inclines to mysticism by the problem of knowledge: here a particular soul, in the way of ecstasy, rises to the ineffable One.

3. At the beginning of the New Age

After what Middle Ages (Aquinas, Bonaventure, Duns Scotus) has brought – when the matter is of the whole being – were mostly speculations in terms of matter and form, of substance and accidents, of essence and attributes, until Roger Bacon (1214–1294), who in the work *Opus maius* entered into the trace of a profound truth – to which particularly our time has given the right:

> The door and the key to science is mathematics ... and it was always preferred to other sciences by all honest and wise men. Because without knowing it, one cannot know ... other sciences, nor worldly things. [Bacon R., IV 1 1, p. 97]

But it will take three whole centuries from the Bacon's slogan, until Johannes Kepler (1671–1630), by following Copernicus' heliocentric idea of the structure of the universe, has formulated his laws in astronomy on the organization of the universe. Then, it is the century in which lived Galileo Galilee (1526–1642), who – fortunately – was inclined to experiment, having made a telescope, to confirm cosmic (Kepler's) laws in experience. The movements of heavenly bodies, therefore, "obeyed" to law, to expression, to number, and it is an important aspect of being which now has obeyed to calculus. Many natural discoveries fallowed too: in biology have been discovered the cells (Hook, 1665), the circulation of blood flow (Harvey, 1628) and others, that led soon to the understanding not only of human as a (natural) "machine", but also of community-state, this time as a (artificial) mechanism (Hobbs). And a further development of chemistry, physics, mathematics that took place, largely supported such a hypothesis, by "contributing" to a more "precise" picture of the world which discovered itself before man on that basis.

But whether man has found indeed his way to reality, even if he succeeded in "discerning" some (large) parts of it? For nevertheless neither is he a "thinking machine" (Descartes), nor physical laws could be legally applied to the field of psyche, society, history, as he is not (till the end) a rational being too. If we leave aside the philosophical problem of the (im)possibility of knowledge of the "being in itself", since as an assumption here serves that the being itself is a rational (knowable) creation – and what, in principle, in no way we can know – we are left to seek the certainty of knowledge and find it in the (relative) permanence of the way in which parts of being enter the composition of a whole, as well as in such a relationship which different wholes (bodies) achieve among them – taking into account as well that it occurs in a space which is infinite and over equally infinite time.

For when Hook, say, finds that organisms resemble a honeycomb, whose cavities coated with membranes he calls "cells", and it was discovered that each of them contains protoplasm, cytoplasm, etc., we can say that their structure on the micro level is that of parts and whole, just as on the macro world cells build tissues, tissues – organs, and organs – organisms. A similar thing is with atoms, and compositions of atoms and so on.

4. Empiricists, rationalists

John Locke also expressed himself in terms of "sets" when, speaking about the origin of all ideas we have, he differentiated two types of them, namely: *ideas of sensation* and *ideas of reflection*. They all in turn allow various "subsets", in accordance with some criteria. So we talk about ideas that originate from a *single* sense and of those originating from *several* senses, about the so-called *primary* and

secondary qualities, or about *simple* and *complex* ideas, which in turn are, say: *ideas of substance*, *ideas of modes* and *ideas of relations* and so on.

But this time too Locke did not stand up for analyzing and codifying deeper and "from inside" the elements of a true knowledge in the realm of ideas and the like – what will become in the twentieth century of primary interest, say, in analytical philosophy, or in logic.

In another empiricist philosopher, Francis Bacon (1561–1626), we meet that a great "renewal" (*instauratio magna*) of sciences must deprive itself of many false notions, or "idols", fallacies and prejudices in thinking, because "the mind resembles those uneven mirrors, which impart their own properties to different objects, from which rays are emitted, and distort and disfigure them."' [Bacon F., p.347] This "'hygiene" of mind would permeate sense experience, cleaning it of various ghosts and illusions, which are owed either to any kind of characteristics of species (the nature of human being), or to a particular "structure" of each individual, or to an uncritical acquiescing to authorities and others.

On the same road of capability of an integral (as strict) knowledge of world, in general, Descartes followed the path of doubt to a dramatic statement: *Cogito ergo sum*, finding that our knowledge is as certain, as the existence itself. And that all is true we (intuitively) perceive as *clear* and *distinct* and what the rational power ("common sense") in us reach by induction and deduction, by following four rules of "method", formulated by him. Moreover, the edifice of philosophy itself, as a rigorous science, finds its starting point in his *cogito*, since now on a rational, as a purely mechanical basis, one can draw conclusions about all there is. For man himself is, after Descartes, machinery too and so on.

For Spinoza, in turn, a single substance, differently named as *God*, *nature*, *causa sui* et al., "disintegrates" in two attributes for us: *thought* and *extension*, "divided" in infinitely many modes and infinite number of ideas. Then to each mode (fact, state) in the nature corresponds an idea and *vice versa*, as here "the order and connection of ideas are the same as order and connection of things," according to the doctrine of *psychophysical parallelism*, which he explains. In addition, it goes in favor of a complete knowledge of the world too, where otherwise, according to him, all is predetermined for the last time.

Of other rationalist teachings about substance, that of Leibniz differs in several aspects. Here the basic unit is the *monad*, which is of a spiritual and not of a material nature. An endless variety of monads populates the universe of what exists and, seen from the outside, they are impenetrable, and (in themselves) closed units. Each of them is a microcosm, for what the universe contains is the entire content of monad, while one in it from the beginning laid principle, called *appetitio*, has just as goal: to clarify its own content. Through a gradual path, from the vague, so-called "small perceptions" (*petites perceptiones*) to conscious representations or *apperceptions*. The clarity of representations is, otherwise, the only thing differing two monads between them, because they all share the same content, which is the content of the world.

Thus, the whole being appears to Leibniz as a pyramidal edifice, on the top of which is the *monad of all monads* (*monas monadon*), God, for he most clearly perceives the wholeness of all existing in himself, and at the bottom – the so-called *bare monads*, which are the least clear. Leibniz called monads *simple substances* (or souls) too, which enter the *complex substances* (bodies et al.), and the image of being, according to him, is that it is a set, or a conglomerate of parts, each of which is either a monad, or a set of monads. So the whole universe, according to him, is animated,...

And that a monad is "equal to itself", as a unique and unrepeatable degree on the same line of clarity of representations, here is a "metaphysical picture" of what, otherwise, have brought different theories of personality (subjectivism, personalism, etc.) in our time. Because it's by the end peculiar human genetic record and his psycho-physical structure, to what the experience many times gave the right. It agrees in so far with the existence of *causa finalis* too, which, after Leibniz, rule the souls, and what in a "non-metaphysical" sense, of course, frees up space for man's individual action. However, although discrete units, Leibniz's monads are equally ruled by the principle of *continuity*, since the "nature does not make jumps" (*natura salta non facit*), what is confirmed by the mentioned small perceptions, so that his view of the whole of being is that it is unity of discreet and continuous. And when it is about development, as emergence of something new in the nature, it does not exist as such, by Leibniz, but the matter is always of "preformism", as a transformation, or "unfolding" of what has already been given in embryo. Hence one concludes, "the present state of it is big with the future" (Leibniz, p.22), as well as "every state of a simple substance is a natural consequence of its preceding state". (Leibniz, *ibidem*). In addition, all "living beings are organically connected with the inorganic matter" (Leibniz, *ibid*) – all of what goes in favor of the possibility of knowledge of the world to a high and the highest degree, by following the same ontological principles in this epistemological area too.

Following our primary interest in the views of philosophers over centuries on being and knowledge to recognize elements of Cantor's theory of sets from nineteenth century, we find now that an infinite multitude of monads, as discrete units of force, correspond to the notion of countable set of Cantor, and that they occur continuously – to the notion of uncountable set. Then, Leibniz's conception of complex substances as "aggregates" composed of those simple (or monads), is like Cantor's definition of set as any "whole of ... defined and different objects of our perception or thought", as, for example, his *monas monadon* would bring an amalgam with the notion of set of all sets. Also, parts (wholes) of animate and inanimate nature: bodies, plants ... would be particular power sets (or sets of subsets), what equally would be the case with the set of representations of particular monads etc. But, here are not codified the most diverse ways in which (simpler) parts are included in the complex ones, what essentially just brought set theory with the notion of "operation" or "function" over sets, and especially with the notions of "cardinal" and "ordinal" number.

In David Hume, we have that with his (destructive) insight into the notion of causality he denied the necessity of succession of consequence from the cause, finding that we are only used to expect that something will come after something, believing that it will happen as well. For what is *causality*, after him? A complex idea, made up of three simple ideas: of *spatial contact*, of *time succession* and of *necessary succession*, where, according to the empiricist slogan that "nothing is in the intellect that was not first in the senses", each of them must have an appropriate impression. However, while spatial contact and time succession leave clear impressions in the senses, it is not the case with "necessary" production of a consequence by cause. Nor can this relation be carried out through reason, by any analysis of the first concept, until we get the other one. Because, it is said, from the idea of water does not follow that we can be drowned in it, or from the idea of fire that it can burn us. (As it is known, from that place proceeded Kant, in creating his famous three "critiques", within the so-called system of "transcendental idealism".)

Looking for an analog with reasoning in this area among the operations of set theory, we find it in the following circumstance: any cause and any consequence can be represented as two sets, which are already thus well-defined objects, while to a causal relationship would correspond a mapping (or a function) from the first to the second set. But, as it is well known, a mapping in this science is determined as – arbitrarily – joining of (every) element of the first set to an element of the other one, and this relationship is not, in general, the relationship of a necessary connection. Hume, likewise, was against metaphysics as a science ("Does it an abstract reasoning", he asks of any such book, "then in the fire with it, because it can contain only sophistry and deception!" (Hume, p.256), following his primary interest for clear and distinct, but leaving equally place – to an unknown and secret. The latter is illustrated by the notion of causality, according to him, because he could not succeed to subsume it neither under a sensual, nor under a rational element, and what its set equivalent finds, as we have said, in the concept of mapping, and the former one – in the belonging of an element to a set, when it is well-defined as object.

5. From Kant to Hegel

How Kant has "fertilized" the "deadlock" (Russell) between empirical and rational in the knowledge, and what is in it that enables to facts of experience necessity and general validity, according to him, when they do not have an objective basis in experience? By revealing "synthetic" *a priori* forms in our mind, to which he assigns to be that "bridge" between cause and consequence, and which, precisely, through our subjective power, in a necessary way, organize experience. Here come *space* and *time*, as pure (synthetic) forms of sensuality and twelve categories (*quantity*, *quality*, *modes*, and *relations*) as forms of understanding. These are conceptual forms of *synthesis* (of a subject and of a predicate in knowledge), as that new in it, otherwise, different from purely *analytical* forms, just as relations between concepts. On them is based therefore the whole knowledge about reality we have, as constitutive forms of it, just making assertions we do in experience to be necessary

and generally valid. In that place Kant speaks too about so-called "transcendental apperception" etc.

"Thoughts without content are void; intuitions without conceptions, blind" [Kant, p. 75], and, Kant says, on it are based all rational notions in experience, and if we should find in set theory where do reign – similarly – constructive laws, it would be so-called "countable sets". The power of them is that of rational numbers, and of these in turn – of natural numbers, which, otherwise, can be represented with only two signs 0 and 1 (two states, etc.) – and it certainly would be the case with the (rational) notions we reach in experience, because they all make a discrete (i.e. countable) totality of objects. So we would succeed, in sufficiently exact way, (Kantian) terms related to space, time, quantity, relation,... to express in the language of numbers – or through two mentioned symbols 0 and 1. On what otherwise is based the application of computers in informatics today, until the possibility to establish artificial intelligence itself.

Kant equally speaks about the power of reason in us, which gives rise to transcendental ideas – now turned to the rational knowledge, precisely as to the totality of conditions, as that chain connectivity of factors without end, which determine a phenomenon in experience. Here object, therefore, is not given in experience and the understanding cannot create a notion of it, but the matter is of an idea deprived of objective basis, such as: *the soul*, *the world*, *God*. Those "pure notions of reason" generate a greater unity of knowledge than the categories of understanding in experience, having a practical and heuristic character. But, although these are purely subjective principles, the metaphysics takes them as an object of knowledge, and hence it could not survive, according to Kant, as a philosophical discipline. As abstractions, they help an organization and a greater unity of knowledge, (possibly) its anticipation in the experience etc., "directing reason towards a particular purpose". Such notions-ideas are in logic: the law of identity, in mathematics: the set of sets, in physics: the universal law of nature, etc. The result is that by dealing with these ideas, in order to provide them with necessity and general validity, the understanding falls into insoluble "antinomies of reason", etc.

As for the analogy with the set theory, to creations of reason "correspond" uncountable sets in it – and then the real (cardinal and ordinal) numbers in mathematics – which occur, otherwise, as limit of (infinite) sequences of rational numbers. Just as continuous values can be presented by series of everywhere discontinuous (discrete) values, doing the discourse about them to be, ultimately, achieved in terms of rational (and then natural) numbers.

After Kant, in the history of philosophy, we have in Fichte that "all being is conceivable only as an object of reason" (after Windelband), since it is not for human to contemplate, but to act and it is not a fact, but an act an original postulate of consciousness. So starting from the "self" as a basic category in the theory of knowledge, he rises above the passive stance of Kant's idealism to an active role of subject in the process of knowledge, and what we learn, according to him, is rather "the being for us", than "the being in itself".

In favor of the possibility "subjective to come up to objective ... and to agree with it" – that is, of the principal knowability of world – speaks Schelling too, for whom the nature and the spirit are only two ways of occurrence of the same absolute, so that the laws of the objective to be "originally identical" to the procedures of our consciousness. Because "Nature is visible Spirit; Spirit is invisible Nature", according to him, etc.

Finally, Hegel's "one and many" is *absolute mind*, or *absolute idea*, as in fact a dialectical totality of objects, that knows only its own self-creation, as the acquiring of consciousness about oneself, just in the *notion*. His absolute is the unity with itself in difference and it is not merely indifferent given thing, as in Schelling, but it knows movement, development, as it is permeated by "rationality" to the end too. The being is, therefore, for Hegel, intelligible without the rest – or: "The primary essence of the universe has not force in itself which would be able to resist the power of knowledge," as he spoke to students at the beginning of his lectures. However, his *thesis* and *antithesis* pass (schematically) almost an "identical" path up to the "unity of opposites" in *synthesis* – from a "pure nothing," at the bottom of the pyramid, to the "Prussian state", on its top – not explicating the ways in which content-marks of parts constitute the concepts they make. And, possibly, to accomplish in this way a certain order among them, and certainly to set them clearly and distinctly. Otherwise, his *panlogism*, as a whole, is a "triad of triads", each of which (except the first one), contains a simpler triad in itself, etc. – what allows it to be presented, say, in the way of a *graph*, in mathematics. The structure of being is, therefore, that of a "mosaic" or of a "collage".

A certain parallel with the theory of evolution, as well as the emergence of what is new, we suspect in Hegel's teaching, inasmuch as all that survives in the nature is a result of adaptation of an individual to the environment, as a specific "exchange" of elements between it and the environment.

6. Epilogue

Following this arc in the theory of knowledge in the West, from Thales to Hegel, regarding the possibility of a comprehensive knowledge of the world, and then (possible) ways of a person, of a community of people to find favorable outcomes for themselves, before the temptations of blind forces of the world, if we have overlooked the skeptical arguments on this path (Gorgias: "Nothing exists", "If there exists something it cannot be known", ...), the theological influences upon cognition, such as inspiration, illumination, ... or metaphysical speculations, we find that philosophical and scientific thought in the West, have reached some moments that were a positive contribution to a true knowledge (most generally were accepted, confirmed in experience, etc.). They can be now (briefly) "listed" and pointed to some inherent problems in the way of the following sketch of the universe in terms of its knowability:

i) the world being is through senses, through understanding, reason, intuition ... to a high extent cognizable "for us" – if not "per se" and by the end – from one to the

other end of it, as in every part is reflected the wholeness ("one and all" – Parmenides and others.);

ii) the knowledge gained by senses is (only) more or less reliable (probable), being confirmed (or rejected) in the experience (Plato, Locke et al.);

iii) on the way of knowledge we build notions (propositions, conclusions), and we ask of them to be clear and distinct (Socrates, Descartes);

iv) the problem of (actual) infinity never stopped to be a puzzle and it has not been found a key to approach it;

v) although almost since the beginning of the philosophical thought in the West, Pythagoras postulated the thesis: "All is number", until the advent of computers in the last century, we do not meet successful attempts to "approach", or to "look up" some *large parts* of the world being in number. Insofar as in our time the (forced) efforts are made just to construct a machine that would rule the intellectual power, imitating thus the work of human brain on the same basis, basis of number.

7. The Being and the Set theory

Namely, on that way we should conceive, first, a structure of world being and, "settle" it by the structure of numbers, and then determine, in accordance with it, the very concept of actual infinity (of continuum). As it did Georg Cantor, in the twentieth century, in the set theory he has created. Here his definition of the notion of *set* is that it is: "gathering together into a whole of definite, distinct objects of our perception [*Anschauung*] or of our thought", [Cantor, p.46] that he calls *elements* of set. Thus "element" and "set" are now together at the beginning of this exact science, to what once in the Antiquity served the terms "one" and "all". We have then that – consequently – (in the same way) is defined the notion of *subset* of a set, what suffices Cantor's definition to "reveal" an image of being as a *mosaic*, or a set of sets (of elements), each of which, for its part, is the same.

Then where numbers find their place in the ontological picture of the world that brings this theory? Just when it should be determined the "number of elements" of a collection – as its "cardinal number", i.e. be indicated the place of an object in it – as "ordinal number". In the first case, for example, it is said: "We will call by the name *power* or *cardinal number* of *M* the general concept which, by means of our active faculty of thought, arises from the aggregate *M* when we make abstraction of the nature of its various elements *m* and of the order in which they are given" [Cantor, *ibidem*], and similarly it is determined the ordinal number. It favored Cantor's intention to construct (to define) *continuum* – otherwise, uncountable set of points on the right – as a set of all orderings of an enumerable set (*axiom of choice*), and when it should be still showed that there is a set whose cardinal number is (strictly) greater than enumerable set, and (strictly) smaller than the continuum (*the continuum hypothesis*).

But – alas! – it has been shown later that both the axiom of choice [Gödel, 1940] and the continuum hypothesis [Cohen, 1963] are independent of the axioms of set theory, what could be interpreted as: there is, indeed, a "terrestrial", a "phenomenal"

mathematics, which takes into account the continuum hypothesis, and the other "celestial", "noumenal", where it is not the case. The actual infinity, therefore, slinks off intuition, as continuity, as continuum, because – like the *apeiron* in Anaximander – they are living creations, never equal to themselves, uncatchable in the concept.

The achievements of Cantor's theory of sets is equally that the infinity is not a "homogeneous" or "linear" value, but reveals a relief image of a highest complexity, that there are complexities of different levels (countable, uncountable, etc.), and even there are arbitrary (infinitely) many of them. As well as he has replaced the points on real line (with enough reasons) by expressions, and thus visible spatial entities he has replaced by number expressions as abstract (arithmetical) values. Moreover, his set theory gives right to metaphysics too: "I think that metaphysics and mathematics should be in the interdependence" [Cantor, 1970], he will say etc.

But in the sphere of phenomena, set theory failed to reach "that living wealth of the world" (*Russell's paradox*, etc.), nor in such a way mathematics could be reconstructed as a complete and consistent theory, so that formal axiomatic systems have been made (Hilbert and al.) in order to be "imitated" as an intuitive area, in order to demonstrate in that manner those metamathematical properties of it. But, this time too, an intuitive domain has appeared to be "more abundant" of each formal expression about it, because Gödel finds out intuitively true propositions, but formally unprovable in the system. [Gödel 1931]. What was followed by his proof [Gödel 1933] that consistency of arithmetic (mathematics) cannot be proven by means of what can be formalized in it, so that, in addition, Church and Turing [Turing 1937] will show that there is no procedure by which it is able to decide whether a proposition is provable in the theory (theorem), or not (*Entscheidungsproblem*, Hilbert in 1928.)

It is not possible, therefore, to mechanize methods in mathematics by the end, in the manner of formal systems which imitate them, and in that way it is proven that the timeless essence of number is not crucial in achieving the exactness to which the science tends to.

8. Turing and Computer Sciences

And while once – with Riemann, Lobachevski, … – the *evidence* was rejected as the basis for proofs in geometry, and then in physics, which relied on it, now with Turing we have that what eludes to the formalization of mathematics, and what axioms fail to reach, is by its nature, *precisely geometric*. Namely, through an analysis of acts of thought that a person performs in solving a problem algorithmically, he saves in a table the elements obtained in that way, finding them to be purely mechanical by nature, with which can proceed a machine too. It brings his idea too of s. c. "A- machine"[1] [Turing 1938] (called out in 1937 "Turing machine" – Hilbert), as an ideal device to whom belongs an (infinitely) prolongeable ribbon, on both sides, divided into fields. The machine distinguishes finitely many *symbols*, and

[1] It is what we call a "program" in informatics today.

a finite number of *states*[1], and its work consists in printing, or deleting symbols in a field and in transition to a new state, in moving left, right, and reprinting, or deleting symbols and the like. It has, therefore, for the "effect" between two (discrete) moments of time (only) to do a certain transformation of symbols of which one started, whereas Turing (Church, Kleene) convincingly argues that every algorithmic solvability of a problem finds its "equivalent" in its computability in a way of Turing machine (*Church-Turing thesis*). Now, instead of things, one speaks about numbers and so on, so that one of his arguments would be, say: "I shall suppose that the number of symbols, which may be printed, is finite. If we were to allow an infinity of symbols, then there would be symbols differing to an arbitrarily small extent" (Turing 1937-8). Later it was shown that intuitive computability (solvability, decidability) can be specified in more – otherwise, equivalent ways – as are: Church λ-calculus, Post canonical system, Smulian elementary system, Herbrand-Gödel-Kleene general recursiveness etc. Moreover, he conceived an *universal machine* of this type[2] [Turing 1937], open to each algorithmic computability, in general etc., so it can be said that he specified in this way precisely the concept of *calculus*, and did it with the algorithmic calculability too (with the notion of *algorithm*).

Thus, when he wanted to move the limits of computable, in order to prove improvable Gödel's "type" propositions, Turing has come on the trail of what "programs", "software" and "hardware" are in informatics, and what knows any computer, mobile phone today. Over this, in his programs he looked for a place for those aleatory processes in the nature (in physics, in biology) not subjected to (mechanical) laws, and what led him to investigate the structure of organic forms, and of intellect. So he was occupied – like calculus – to define exactly the notion of *intelligence***,** as a substance not depending on the substrate (living, non-living), that would make possible to bring it from the outside in it (*artificial intelligence*). He conceived the game of "imitation" as well (*Turing test*) in order to do (not) distinctive answers that come from machine, of those given by a human being and so on. He asks: "Is it theoretically possible for a finite state digital computer, provided with a large but finite table of instructions, or program, to provide responses to questions that would fool an unknowing interrogator it is a human being?". [Turing 1950].

But while the *configuration* (defined by a finite number of symbols and states) of a Turing machine, as its definition, was a sufficiently simplified concept, the concept of an artificial intelligence emphasizes (so far) its highest complexity – eluding formalization – being, rather, approached from particular aspects, determined usually by (practical) circumstances in which it is expressed. What is in the intersection of all conceived definitions certainly is an inner urge of each entity endowed with that power to preserve its own being in the world (environment), as different over again, but to realize a harmonious relation with it and the beings

[1] A double role of symbols is: to be transformed and to indicate the next step in the work of machine.

[2] Having solved in that way the aforementioned *Entscheidungsproblem* (as algorithmic solvability of an arbitrary problem) from Hilbert program. It corresponds to a digital computer today.

surrounding him. And not only it. He would like to see the world different than it is, to change it and in that sense he is guided to consider strategies and to assess consequences. Here it can be applied a specific *computational logic*, which includes, however, the classical logic (based on the connectives: *not*, *and*, *or*, *if ... then* ...), but which brings also procedures for reducing *goals* to *sub-goals*. Like symbols in Turing machines and their double nature: both to be altered and to indicate the next step in the work of machine.

Computational logic, after, has to take into account what indicate many psychological experiments today, namely an individual, in the environmental conditions, tries rather to adapt to circumstances, than to follow any "classical" and "generally accepted" logic. In that sense it develops different strategies and looks for one which is the best (with the chance of success), while giving the "parts" of its act different "weights" of probability, realizability, etc. So it can be said that those discrete states of machine language ("there is electricity" and "there is no electricity", etc.), or equally rudimentary language used in the software (programs) in informatics today, relying on classical logic, essentially, are a really powerful tool in cognitive theory.

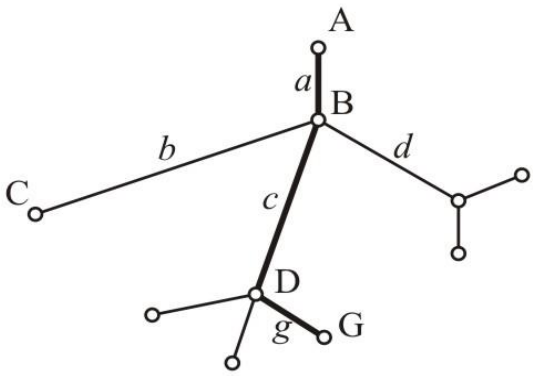
In favor of it are arguments of cognitive theories (functionalism, behaviorism,...), namely what practically is of significance for a mental state of a human (aspiration, pain, ...) are not so much his biological (psychological, etc.) prerequisites, as the role belonging to them within an operational system of behavior. (By the example of an aircraft which flies, and nevertheless does not share biological properties of birds.) Ramsey has showed, in the same sense, that it is possible to reduce to a certain canonic form, all psychological states and processes (but arbitrarily others too), for example the sentence: "Bodily injury causes pain; pain leads to anxiety; pain produces effort not to fall into such a state" and so on. Now if we replace different mental states by variables and apply existential quantifiers to them afterwards, we get the expression $\exists x \exists y \exists z(x$ causes pain & x leads to y & x produces $z)$, and this expression does not contain any mental labels in itself.

An expression, which is, of course, easily translatable into the language of Turing machines, or programming languages, as well as into each of formal language in general. Because, if we enter into certain commands of programming languages, we find that they follow the same alphabet of Turing machines, in that they are not more than a transformation of certain groups of symbols between two discrete moments of time (commands: *input*, *if ... then*, *procedure*, *repeat*, *end* etc.). After it, with sufficiently informal reasons, it is argued that all algorithmically procedures are equivalent among them, what suggests one and the same "range" in their epistemological (cognitive-theoretical) sense.

9. On the language of *nodes* and *branches*

It is interesting that all complexity of drawing conclusions (decisions, procedures etc.) from premises in this area "grows" upon one and the same matrix-base, for example, "nodes" and "branch", in the language of graphs, where nodes

correspond to sets (Turing machines states, software, ...), and to branches operations (functions, subprograms, sub formulas, ...). By the example of the scheme in the tree form:



where in each node: *A*, *B*, *C*, *D*,... appear one or more possibilities: *a*, *b*, *c*, *d*, ... which then, according to some rule (program) would be selected: for example, *acg* (*ABDG*).

In favor of it is the fact that ontological picture of the world – from microcosm to macrocosm – reaches such a schematic figure, what illustrates, in a series of examples, Georges Chapouthier [Chapouthier 2013]. The mater is of two principles: *juxtaposition* and *integration*, upon which rests the whole complexity in live and inanimate nature, according to him, and can be recognized in the "memory, consciousness, language, drawing, music, technical objects, mathematics, social structures, dialectic, and ethical stances". [Chapouthier *ibidem*, p.201]. According to the example of a "necklace with identical beads" [Chapouthier 2009, p.3], in the first case, i.e. a "necklace with beads of different colours and shapes" [Chapouthier *ibidem*], in the second one. Now these objects, as wholes, are parts of a new entity, following the same principles, and both times it is so that "the functioning of the whole does not cancel the autonomy of component parts". Etc.

We have marked, in the same way, by the words "conjunction" and "fusion" (and the opposite processes by the words: "disjunction" and "melting"), when, in an ontological sense, we have interpreted potentially power fuzzy set, which has the structure of category in mathematics. [Tasić 2010]. Then – in the language of fuzzy sets and using an appropriate fuzzy logic – we indicated that by using, say, methods of optimization, one can realize a whole (an action, an outcome), which is, from some point of view, the most suitable for us (the most likely, etc.).

All this has found a reflection in the notion of "artificial intelligence", whose proponents "study and design of intelligent agents", as those which "perceive its environment and takes actions that maximize its chances of success". What "includes reasoning, knowledge, planning, learning, communication, perception and the ability to move and manipulate objects" [after Poole, Mackworth, Goebel 1998], to what

should serve methods of logic, mathematics, information theory, through robotics, cybernetics, to neurology, or even the economy.

10. Mathematics as ontology

The original (Aristotelian) definition of science of ontology is that it is a science of "Being as such and the properties that belong to it" [Aristotle 1003 *a*], as well as the ancient (Pythagoras, Plato, ...) looked at Being as to a whole of "one and many". But if we mention Cantor's definition of set (1871–1884) we see that it was derived in sufficiently general – mathematical, but epistemological (philosophical) terms too – what does possible the conclusions about Being and knowledge to be equally performed in terms of this mathematical theory. This definition is, namely: "A set is gathering together into a whole of definite, distinct objects of our perception [*Anschauung*] and of our thought…" [Cantor, p.46], which was extended in this century, for example, to the attitude of Alain Badiou: "Mathematics is ontology". Certainly not in the sense the world is populated by mathematical beings, but that "during its entire history mathematics expressed what is *sayable* (*dicible*) of being-as-such." [Badiou, p.11]. Inasmuch as in the mathematical branch of topology – which in its name has the word "topos" (place) – plenty of results it achieves can "immediately" be translated into the language of ontology, being made through it the very expression of Being.

Namely, first, we have that in the way of set theory are interpreted classical parts of mathematics: algebra, analysis, geometry, ... so that – with some difficulties – the "Cantorian paradise" (Hilbert), on the line of generalization, led to the concept of "category" in the last century, which by name, would refer to the importance of this word in Aristotle's doctrine of being. In fact, when proceeding from that that both classical and non-classical theories are based (only) on the elements (sets) and relationships between them (functions) – as well as some relations that accompany them – the words "element", "set" are replaced (better: generalized) by the words "object", and the word "function" – by the word "morphism". The two mentioned relations were: *identity* and *associativity*, etc., and so this meagre variety of concepts is found in the definition of the "category", to be shown afterwards that category theory is a sufficiently identical (common) universe of reasoning in a series of mathematical branches. And not only in them. For instance, in the topology the objects are topological spaces, and morphisms continuous mappings among them, in linear algebra the vector spaces are the former and linear mappings – the second one and so on. In this way, now set, multitudes are not determined by properties of elements that make them, but by their overall relations with other sets, the wholes.

In ontological terms, the case is also with each individual: he is essentially determined by his relationship with other individuals from the "category" they belong together – as in an evolutionary sense he is a product of the environment. The word is about, says F. Jedrzejewski: "Not to develop an ontology in terms of the *logic of worlds*, but a topology of the universe, of space whose texture is so diverse to give

more consistent meaning than the one done possible by a logical shell". [Jedrzejewski, p.6].

Thus, the topological concepts gain here a primacy over logical ones – the notion of environment over the notion of implication – and so on the Being and the One it is spoken out this time from the depths of topological objects and their morphisms, as a belonging logic they should to order. Later, the concept of *topos* has received an exclusive place in their organization and classification of category-worlds, as well as their universal importance facilitated the understanding of identity and diversity, of duality of Being and the One, of their functionality, univocity... It would be a particular way back to Heideggerian "forgetting of Being", other than addressing the idea (Plato), the substance (Aristotle), the monad (Leibniz),...

The notion of topos. By the notion of topos is formalized a strong intuition in mathematics, in so far as we can test so the properties of objects themselves, not only those from their relations with other objects. Just as it is in the set theory. It is a particular (Cartesian closed) category, with a special morphism (arrow) – the so-called "subobject classifier" – which makes possible for each set to establish whether it is a subset of a given set or not. Its discovery (about) 1960, apart that it contributed to solving problems in algebraic geometry, and elsewhere, has confirmed a highly synthetic power of this concept in the "unification" of mathematical knowledge, but it turned out to be a powerful tool for the interpretation of Being too, in the methodological-epistemological terms.

We have after that the topos structure determines its belonging to logic, and it is, in general, the intuitionistic one (here, it does not hold the principle of the excluded middle). But one finds that if the axiom of choice is valid on the topos, its logic becomes classical. [Diaconescu, 1975]. That is a far-reaching significance of this proposition (theorem), since it affirms that the texture of a topos acquires its supremacy over its logic. And when the axiom of choice applies on it, then all infinite subsets have the structure of the One and Many, etc. So that Grothendieck showed equally that in topological spaces, open sets provide more informations than the points they contain, or that the environment to be "more important", than the belonging elements.

Alain Badiou will develop an ample reflection on the Being and truth in his works *Being and Event* and *Logics of worlds*, in the closest connection with the set (topological) terms, just on the trail of thesis that back to Greeks the truth of Being is bringing rather by *matheme* than by Heideggerian *poieme*.

On the universal. As a notion, the "universal" is also subordinate to the notion of "categorical", since it can be derived from it. "Category of all categories" it is both "local" and "global", identical for all objects and morphisms. According to Jedrzejewski: "The universal comes from the categories and is not a form from outside, which would be materialized in particularities". [Jedrzejewski, p.116]. As a logical concept, its negation is not individual, particular, but by topological properties that possesses, it is rather recognizable on both sides: from natural to social sciences, as well as in everything surrounding us.

On duality. The principle of duality is posted in the basis of this world – both in what exists and in what becomes an expression about it ... According to Jedrzejewski, "to understand the origin of dual forms represents an important stake for the science and philosophy too" [Jedrzejewski, p.88], because the dual concepts in the science of being are One and Many, actual and virtual (they do real), the identity and differentiation, ... It seems that this trait the Being has carried off on it since the Big Bang[1] – which, among others, makes (easier) understanding of one member of the dyad when we know the other one. In so far is more truthful Badiou's slogan that mathematics is ontology, for if mathematical and physical sciences bring the truth about the One, and the One is dual with many, as Being,[2] the science of Being (ontology) would realize the relation of identity with them.

There are numerous examples of these concepts in physics, such as particles and waves, electric and magnetic fields, matter and antimatter, ..., or of Faraday and Amphere laws *et al*. In logic those concepts are: *and* and *or*, *some* and *for every*, or DeMorgan's laws, and in geometry – point and right line ("If a right line passes through two points", then "two lines intersect at the same point"), sides and angles. Of solids, tetrahedron is dual to itself, the cube is that to an octahedron, a dodecahedron – to an icosahedron, as are theorems of Pascal and Brianchon in projective geometry. And that's the case with many examples of mathematical objects, operations, assertions and theorems. Or, the objects have their co-objects, properties co-properties, diagrams co-diagrams and the like – so that it is just a way, for example, the objects of a theory to be classified by the objects of the other one.

The concept is of an exclusive importance in a number of theories and problems in mathematics (groups of Grothendieck-Galois *et al*.), in the string theory in physics, and so on. Let us say, finally, that this Janus' face of things and beings is recognized in epistemology too. Here the truth reveals a double character: it is *aletheia* as truth of becoming and *adequatio* (Heidegger), as truth of the One, etc. As to categories, in the category theory, if you keep objects and change the direction of the arrows (morphisms sense), one obtains a category dual to the given one. The same may be said too that for any graph there is a graph dual with it.

It should be said that many of couples of terms that "come together" (opposite, contradictory) are not mutually dual. The case is with the concepts of discrete and continuous, analytic and synthetic, necessary and contingent... "What distinguishes the opposition from duality is that the latter is rather an identity, than a difference", says Jedrzejewski. So to dualities it belongs an agreed and "natural" existence – by example of particles and waves, electric and magnetic fields,...

On the factoriality. If the Aristotelian notion of relation finds its analogue in the notion of morphism in category theory, it becomes generalized to the notion of functor, which brings the connection between objects of different categories – by

[1] Plato's dialogue *Timaeus* brings the myth of origins of the world from the ratio of the *Same* and *Many*.

[2] For Badiou the member dual with the Being is "event" (*événement*), and for Sartre – "nothingness".

transferring the properties of one of them in another category. A functor, then, corresponds to the "analogy" in the intuitive area, which as a methodological approach is, otherwise, widely used in all domains of mind: from science, through religion, to literature (the concept of metaphor). Because when the parts of Being confirm by it their "identical" structure, or an "analogous" functionality, it only testifies about the unity of Being and our increased power to know it. So we talk about the supernatural world "according to the figure" of the world down, as four types of statements in logic, Apuleius represents with spatial (planar) figure – by "logical square". "Porphyry's tree" too during the Antiquity and the Middle Ages served to easier understanding, as Aristotle, in *Nicomachean Ethics*, say, the moral terms will compare with categories of mind in the same sense, and so on.

In physics, Feynman pointed out how "the same equations turn up in the study of several other phenomena: heat flow, distortions of stretched membrane, diffusion of neutrons, irritation of fluid flow, and uniform illumination of a plane" [Maddy, p.335], in order to conclude "that this is due to the unity of nature" [*ibid*]. Or let's give the example of Newton's law of gravity (attracting the masses) and Coulomb's law (the attraction-rejection of electric charge) with the same expression and so on.

Finally, the whole Being, and otherwise, can be interpreted in terms of the One and Many – as those of farthest strongholds, or a common ground of all that arises and disappears in the world.

11. Category theory as a philosophy of mathematics

And more than it: the category theory finds a way, in the same language, to transfer information about the objects making it, and about our perceptions of them – as no other than a particular unity of subject and object. Because – the latest ones do a category as well. It is a consequence of Yoneda's lemma, according to which every category *C* is isomorphic to the category of all functors from *C* to the category of sets **Set**. Then two objects are isomorphic too, if functors associated to them are isomorphic, so that between objects and "viewpoints" to them occurs a relation of equivalence. Subject and object, therefore, are never separated in the knowledge and this mathematical result gives only right to it. And some object will be known from many aspects, if it should be observed in more categories, and if more functors are attributed to it. And so on.

We can continue to follow the language and principles of category theory and try to bring them into a close relation with truths of Being – which, in a philosophical key do, say, Alain Badiou and Gilles Deleuze – by expressing rightly the hope that its "synthetic power" in mathematics will be increasingly extended (applied) to other scientific disciplines, as to the Being alone. Moreover, especially the notion of *topos* favours the creation of "new mathematical worlds" with special properties, which equally does possible different categories and different applications too.

For every scientific theory has its "semantic core", linguistically presented in different ways, and determining *classifying topos* of the theory. Then, the theories are Morita-equivalent if they have the same classifying topoi – a term Olivia Caramello

says, in different occasions, "formalizes in many situations the feeling of 'looking at the same thing in different ways'". [Caramello, *www.oliviacaramello.com*]. Finally, the properties of classifying topoi, remaining invariant in the case of relation of equivalence are those being used to "transfer" the features of one topos-theory into another.

And that some great parts of Being, after the way they are ordered, yield indeed to a possible interpretation in terms of the mathematical theory, suggests O. Caramello's article under the title: *The Unification of Mathematics via Topos Theory* [Caramello 2010]. Some of these parts are, according to her: astronomy, genetics, psychology, linguistics, music...

Namely, to the figure (albeit simplified) of the universe, in the first case, belongs that it is made by stars, around which rotate (finally) many planets. If we compare the latest ones with theories, their classifying topos would be the star, and they alone do an analogue of Morita-equivalent theories, established by the same topos. The fact that all planets perform the same type (of elliptical) orbits, is an invariant feature of the star, to which correspond various examples of Morita-equivalent theories expressing the same property of classifying topos, in the topos theory. Here, for example, Kepler's third law is invariant feature of the star-Sun, and it alone is a "bridge" to transfer information between two planets – because what is true in the case of some of them, would have an counterpart in any other (as well as Kepler's law) [after Caramello 2010].

Quite similarly, if we consider individuals in a society and institutions in it, we can look to the first of them as to theories, and to others as topoi. For example, students go to different schools, but everyone goes to one school, which is thus classifying topos for them. Students from the same school bring analogues of Morita-equivalent theories, that is, theories (classified) by one and the same topos. We recognize here the site too, as a pair of an educational process and student and so on.

And as in topos theory, the classifying topos of different theories can be built from any of them, so the (classifying) school can be determined uniquely from any student (theory), which frequents it. In this way, the school and the classifying topos are "natural environment" for investigation, when, for example, school subjects do invariant properties in it as a classifying topos. In that case each educational effect of the school on a student would be a particular value of that invariant, as a general (abstract) property, while classifying topos (school) would serve as a "bridge" to transfer knowledge to students, but also of knowledge between themselves, as this time Morita-equivalent theories, etc. [*ibid.*].

In the case of translation of a text from one language to another, afterwards, what has (to remain) invariant is a set of some of its abstract properties P which should be preserved (such as context, meaning, poeticity), taking into account the syntactic particularities of two languages as well. So in the case, say, of an expression e in the original language, we find first its "value" P_e in the context P, and then the corresponding value $P_{e'}$ of the expression in the context of language to which we

translate. Here as a "bridge" appears the set *P* of properties to which we have alluded [*ibid.*].

Or in genetics, the human DNA plays a role of classifying topos in the topos theory. Because it does not depend on the human constitution, or age, as the classifying object is invariant too with respect to the manner in which the theory is expressed (axiomatization, etc.). In addition, they both have a role to point out the similarities and differences, transferring the knowledge from one area to another [*ibid.*].

The category theory, therefore, testifies that an intuitive reality – ranging from the nature to human mind – in their various places, gives way to an interpretation in its terms. The fact that the "knowledge" from one of them is transferrable to another category, suggests the possibility of systematic recognition of these circumstances and their studies, to make the solutions in one field to contribute solving problems in another one. In the way of construction of models, of tests etc., while the growth of technology, the development of computer sciences, of program languages and so on, would again and again contribute to the possibility of "intrusion" in events, in phenomena and processes and their descriptions, both in the external nature and in the human sphere, in order to express human needs in relation to them (to control them, to adapt to them, to change them et al.). Otherwise at the macro and micro level, the human succeeds to predict phenomena, even if they almost chaotically take place (in meteorology, epidemiology, economics), as well as if they allow an algorithmic solvability. So if a singular category-monad brings, for example, a category-model of preferably educated individuals, and here is found the classifying topos, we would have the basis for the diffusion of patterns of an exemplary education in the case of an arbitrary number of (other) individuals in the same category etc.

Here is of importance the fact that in the human sphere to subjective characters of human values and acts (moral, aesthetic and other norms) we can attribute different "weights" (as quantities), and then replace them by numbers. Thus, the reasoning in this domain receives, in addition, on the exactness, and it may be done, for example, in the way of fuzzy sets and fuzzy logic. For a fuzzy set of a set sum up in itself the very qualitative labels of its elements, like the values of (fractions) between 0 and 1 – of 0 when it fails, to 1 – when it occurs to a highest degree. Allowing the fact that all in the nature occurs in a way either of *juxtaposition*, or *integration* [Chapouthier 2009, p. 3] of elements-parts – e.g. stones in a mosaic, in the first case, i.e. the hydrogen atoms and oxygen atom in a molecule of water, in the other – we have the possibility, using set operations, to express what's new, as a whole of elements-parts, as well as to make choices desirable from some standpoint. Both in material and in spiritual realm – doing all of it still in terms of category theory.

Otherwise, we have already that subjective in nature is the acquiring of concepts about objects and beings as well, again, no more than as a "set of its essential

marks"[1]. For if someone utters the words: table, house, tree ... each of us will build (at least somewhat) different picture on it, and what would be confirmed if we put forward our own descriptions of these images. However, on the other hand, sentences are sets of words, and to statements will equally belong some (subjective) value between 0 and 1. Therefore, we are able, in the choices (produced freely) to estimate the chance to realize them, in a sufficiently exact way – the way of mathematical models, counts. It would support the fulfilment of that ideal of Leibniz expressed by words: *Calculemus*!, in a case of disputes between the ideas, when it is not known which of them is more likely to succeed. Each record, account,... keeps human memory – instead of a permanent return to the beginning – and do not find room for (otherwise, testified) power of hardware and software today, in the vicinity of what is the core of our being, and that is freedom – would be anachronous.

If he really has to choose, say, the moral values for himself "unconditionally" and "alone" in freedom, would not it be desirable for him to know that search engine Google states (in May 2013., in English) 7.2 billion units for "good", and 0.7 billion for "evil", or 2.5 million for "atheism" and 22 million for "theism" etc? And, to such a large number of samples – with all the relativity in interpretation – "traditionally" so far, no experiment could not count. It also says that people ten times more "mention" good, but evil – the number of which is measured in billions – and nine times more theism, than atheism. And so on. Finally, while we plead here for an eminent place that should belong to the philosophy in an overall development of sciences and the society as a whole – just as thoughtful ones – the test shows again that the index of Philosophy is 304 millions here, while it is for Mathematics 258, for Physics 303, Chemistry 305, or Biology 204 millions. It tells about the place of philosophy itself among the sciences and so on.

Conclusion

So from Thales and the slogan "The world is one", which he has seen through intuition, until machines of Turing and topos theory in mathematics, some (brief) overview of several nodal momenta and controversies in the human endeavor to understand the world around it, would be as follows.

First of all, a happy moment in that way was that Pythagoras the hidden structure of the being replaced by the visible structure of numbers, what has found a reflection later in Plato's "one" (*idea*), which is at the same time and "many" (*particular things*). The double character of one and many does from a part (variable, sense) to watch the whole (unchangeable, universal), but this time in two ways too: as an approximate knowledge (*opinion*), about the first one, and as a true (*reasonable*) knowledge, about the other. Then, early enough in thinking, arose the controversy: "Sophists or Socrates", of the highest importance for further development: namely, whether the knowledge we acquire about the world is necessary and general one (*objective*), or is arbitrary and personal (*subjective*)?

[1] So differently from Socrates' belief that the world can be thought only in necessary and general terms.

Socrates believed in the former, pointing out that it can be achieved in *concepts*, but both times, in Plato and Socrates, the problem remained unsolved until the end: for can the variability of our senses – of which, otherwise, starts all the knowledge – to bring out things and beings as they are "per se", and, in addition, such a knowledge to be general and necessay? Aristotle afterwards the Platonic idea (as something eternal and unchangable, and so far "in the heaven") laid down in the things and beings themselves (now as inconstant and perishable), to perform here both ontological and epistemological, logical and teleological function over them.

Or, in the New Age, Berkeley argued that the very things are "complexes of sensations", while Descartes the cognitive necessity has found in the absolute certitude (*cogito*) of subjective *ego*. And his methodological principle, derived from the notions of *clare* et *distincte*, gained the highest importance in acquiring a certain knowledge about the world in the sciences until today. We should also mention Leibniz, who in the concept of "pre-established harmony" between a monad and other monads, refered to a perfect harmony between what is inside and what is outside, and what deity laid down in all events from the beginning. As well as the main activity of monad is to reflect all what exists, and of the man (who is also a monad) to understand the world. Leibniz speaks about "innate ideas" in us, and so on.

Kant, by length of each of its *Critiques*, will provide an ample evidence that the "nature is according with us... that the human mind and the nature are made for each other". We have so, first, in the theory of knowledge that we found out of things what we enter ourselves into them, so that the *soul*, the *world*, and *God* are only "ideas of pure reason" in us, due to our need these three entities to be embraced as a whole. Then in moral theory it is shown how something willingly and personal may become necessary and general (his *categorical impreativ*), and finally, in the aesthetics, his *principle of finality* has realized the needed arc between the necessity prevailing in the nature and the freedom in the area of morality, by making to tend to the "pure beautiful without a desire to possess the object itself".

Ernst Mah was a physicist who (before Einstein) spoke that we put ourselves the concepts of space and time into the phenomena, and that neither mind nor matter are last instances of reality, but certain "neutral elements of experience", which can occur both as physical bodies and as mental facts. The latter has found reflection in the principles of logical positivists (*Vienna Circle*), which took up a "logical reconstruction of the world" (Carnap), in such a way statements about things to be reduced to statements about sense-data ("theory of description" of Bertrand Russell and others.).

On this track will be Alan Turing two, when he wanted to define the concept of intelligence as an essence not depending on any substrate, and which can be entered from outside into the matter (*artificial intelligence*). (By the example of airplane which to fly does not have to share the properties of birds, or numbers, the essence of which we have not to know to could proceed with them...). But of an utmost importance certainly is the fact that, by an analysis of acts of thought which a human takes when calculating, he displaced the borders of calculable, of number, ...

constructing so called "A machine", and an universal machine of this type, and what will be extended aftrwards to"software" and "hardware" we know today.

Finally, the most exact of sciences, mathematics, in its most abstract branch, category theory, has testified about a particular identity of the world in its different sides. Here, for example, the concepts of place (topos), of environment, of relations has gained primacy in the definition of mathematical objects, over the notions of essence, of logical succession etc., and what finds easy an analogue in the intuitive areas of being. The man is, for instance, a social being, as the laws of evolution too (the adaptation of an individual to the environmental conditions, etc.) are different from the logical laws. Or, by analogy with mathematical theories, having the same classifying topos (*Morita-equivalent theories*), now properties, phenomena and processes in particular parts of reality, can "legally" be recognized in others of them. As well as a mathematical result too (*Yoneda lemma*) "speaks" about the unity of subject and object in knowledge: namely, that every category C is isomorphic to the category consisting of "points of view" of the objects of C and so on.

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