BIOCOSMOLOGY AND CATEGORY THEORY IN MATHEMATICS

Milan TASIĆ

ABSTRACT. *Our intention here is somewhat double one: first, to bring closer a summary doctrine of being to a (special) category theory we construct; and second, to interpret in the sense of the formula of Biocosmology "Bio-3/4", presented by Konstantin S. Khroutski, − the reality of both of them. Having both times in mind mainly the generation of what is new – in nature, as well as in the mathematical area – to point out finally a bold claim: can we find out, within the latest one, in a manner of a "calculus", those optimal choices that a human performs in the sphere of freedom?*

Such a found mathematical structure we have denoted by words "potentially partitive fuzzy set", being arrived to it by using the (classical) concept of fuzzy set and here introduced terms "set replicator" and "potentially partitive set." Therein the supporting logic, as necessary one, would be fuzzy logic, etc.

KEYWORDS: *Biocosmology, category theory, model, potentially partitive set, choice*

1. PRELIMINARIES

1. 1. Fuzzy sets

In order to find a mathematical theory which would serve as a model for a (possible) development theory − as generation of something new in the world of living beings − let us first use the idea of "fuzzy sets" (Zadeh, 1965) and of a "category" in mathematics, as well as the idea of "fuzzy logic" in logic afterwards. For complex processes in the nature and society, or spiritual sphere of a human being know rather degrees (levels, powers, forces, ...) of acting, diverse states, subjective assessments etc., than to permit answers like: "yes", or "no", "belongs" or "not belongs", "true" or "false" etc. and which are expressed by a certain "membership function" on the specified scale.

DEFINITION 1. Let X = {x} be an arbitrary class of objects.

> By $f_A(x)$: $x \to [0, 1]$ let us denote *the membership function* of an object x to the class X, and let us call the set $A = \{f_A(x) \mid x \in X\}$ the *fuzzy set* on X. Therein the value of $f_A(x)$ would be a *fuzzy number* of the object x in X.

Thus the set A gathers in itself all fuzzy numbers, as "degrees of membership" of the object x in the class X, so if, for example, $f_A(x) = 0$, x does not belong to the class and if $f_A(x) = 1$, it does it reliably and to a "highest degree".

NOTE. If the objects of a class are to be expressed in value terms − from some

point of view − and to varying degrees (positive and negative), it is suitable to consider segment [− 1, +1] for the definition of such a "value function" in that class.

EXAMPLE 1. If we denote the age after 70 years as "old age", we should have that people between 60 and 70 years can "gradually" enter this age and to varying degrees to belong to it. So if x are years of life, a membership function of "old people" would be:

$$f(x) = \begin{cases} 0 & x < 60 \\ \dfrac{x-60}{10} & 60 \le x < 70 \\ 1 & 70 < x \end{cases}$$

So a human with 61 years (x = 61) should belong to this set "to a degree" 1, and one with, say, 69 years (x = 69) to a "(higher) degree" 0.9.

EXAMPLE 2. Let N be set of natural numbers and P propriety: "to be close to number 1". Of course, 1 "is the closest to itself", so that the value of membership function in this set at the point 1 is 1, and there is no doubt that this value decreases as the number increases. Thus a fuzzy set A in N would be for example:

$$A = \{f_A(x) \mid f_A: x \rightarrow 1/x, x \in N\},$$

as, in fact, the set A = {1, 1/2, 1/3, ...}.

Otherwise, two fuzzy sets A and B will be *equal* if and only if $f_A(x) = f_B(x)$, for each $x \in X$, and a fuzzy set A be *contained in* a fuzzy set B if and only if $f_A(x) \le f_B(x)$, for each $x \in X$.

Then let A and B be given fuzzy sets and $f_A(x)$ and $f_B(x)$ their membership functions as follows. Three new fuzzy sets A' (*complement*), $A \cup B$ (*union*) and $A \cap B$ (*intersection*) would be defined by the membership functions $1 - f_A(x)$, $\max[f_A(x), f_B(x)]$, $\min [f_A(x), f_B(x)]$, for each $x \in X$, respectively.

Together with set operations fuzzy sets allow the algebraic operations too, such as *sum*, *absolute difference* or *product*. Thus for example *the sum* A + B of two fuzzy sets A and B, whose membership functions $f_A(x)$ and $f_B(x)$, respectively, are defined by membership function $f_{A+B}(x) + f_B(x) = f_A(x) + f_B(x)$ (provided that $f_A(x) + f_B(x) \le 1$), the *absolute difference* |A − B| by the function $f_{A-B}(x) = |f_A(x) - f_B(x)|$, while in the case of the product A·B would be: $f_{A \cdot B}(x) = f_A(x) \cdot f_B(x)$, for each $x \in X$.

Otherwise, the operation is not complementary, for $A \cap A' \neq \emptyset$, $A \cup A' \neq 1$, while $\cup$ and $\cap$ are commutative and associative operations − as are applicable here the distributive laws and the laws of De Morgan.

All it helps us to transfer degrees of membership of objects from parts to the whole, which in various ways (set, algebraic) they realize.

EXAMPLE 3. Let A be a fuzzy set in the set N of the Example 2, determined, therefore, by the membership function $f_A(x) = 1/x$, $x \in N$. Then $f_{A'}(x) = 1 - 1/x$ so that

$$A' = \{0, 1/2, 2/3, ...\}$$

is the complement A' of A in N.

Then for max(1, 0) = 1, max(1/2, 1/2) = 1/2, max(1/3, 2/3) = 2/3 etc., it is

$A \cup A' = \{1, 1/2, 2/3, ...\}$.

Or as min(1, 0) = 0, min(1/2, 1/2) = 1/2, min(1/3, 2/3) = 1/3, we would have that: $A \cap A' = \{0, 1/2, 1/3, ...\}$.

At the same time, it would be: $A + A' = \{1\}$, $|A - A'| = \{0, 1/3, 1/2, ..., 1\}$, $A \cdot A' = \{0, 1/4, 2/9, ...\}$.

So in a fuzzy key we identified processes of formation of something from one or more parts, as well as the way of assessment of an object as a whole, from the assessment of parts that make it, what we take to be an enough real (objective) picture of what comes in experience. It remains to define the notion of "potency" of a fuzzy set, just as a feature appearing in "the sum" of all fuzzy numbers of sets covered by it.

DEFINITION 2. Let $X = \{x_1, x_2, ..., x_n\}$ be an arbitrary class of objects, f_A a function belonging to the class, and $A = \{f_A(x_1), f_A(x_2), ..., f_A(x_n)\}$ a corresponding fuzzy set. Let's call a *potency* (*power*) of A the sum of $f_A(x_1) + f_A(x_2) + ... + f_A(x_n)$ of real numbers, as fuzzy numbers of all objects of X. We denote the potency of A by CA.

EXAMPLE 4. Let $A = \{1, 1/2, 1/3\}$ be a fuzzy set. Potency of this set is CA = 1 + 1/2 + 1/3 = 11/6, while, for example, sets $A \cup A' = \{1, 1/ 2, 2/3\}$, and $A \cap A' = \{0, 1/2, 1/3\}$ have potencies: $C(A \cup A') = 5/6$, $C(A \cdot A') = 17/36$. The example shows that the potency of union of sets is higher than the potency of intersection, which is sufficiently general case, and so on.

1. 2. *Category Theory*

DEFINITION 3. Let us call X, Y, Z, ... *objects* (in symbols **Ob**), and f, g, h, ... *morphisms* (in symbols **Map**). **Map** and **Ob** are *category* C if and only if:

a) for each pair X, Y of objects there exist (arbitrarily many) morphisms between them. If f is such a morphism we write $f: X \to Y$.

b) for each object X there is a morphism $\varepsilon_X: X \to X$ (*identity morphism*).

c) for each $f: X \to Y$, $g: Y \to Z$, there is $h: X \to Z$ (*composition* of *morphisms*). We write then: $h = g \cdot f$.

The conditions a), b) and c) satisfy the axioms:

d) if $f: X \to Y$, $g: Y \to Z$, $h: Z \to U$, then $h \cdot (g \cdot f) = (h \cdot g) \cdot f$ (*associativity*).

e) if $F: X \to Y$, then $f = \varepsilon_Y f = f = f\varepsilon_X$ (*identity*). (Baez 1997).

DEFINITION 4. Let $A \neq \emptyset$. The set **A** is *replicator* of A if and only if at least one element of A has more than one (but finite) occurrence in A.

EXAMPLE 5. Two replicators of the set $A = \{1, 3\}$ are: $\mathbf{A_1} = \{1, 1, 3\}$ and $\mathbf{A_2} = \{1, 3, 3, 3\}$.

Let **A** be a given set replicator. Denote by $\sigma(\mathbf{A})$ an arbitrary element formed by elements of **A**. Let's call σ *singleton* function in A and the value $\sigma(\mathbf{A})$ − a *singleton* of **A**.

EXAMPLE 6. Let $\mathbf{A} = \{2, 2, 3\}$. Three singleton functions σ^1, σ^2 and σ^3 in **A** would be: $\sigma^1: \mathbf{A} \to 2 + 3 = 5$, $\sigma^2: \mathbf{A} \to 2^2 - 3 = 1$, $\sigma^3: \mathbf{A} \to 2 \cdot 3 - 2 = 4$, while 5, 1

and 4 are three singletons of this set.

EXAMPLE 7. Let the (chemical) elements C, H, O be elements of the set A = {C, H, O} and the set **A** = {C, H, H, H, O} is the set−replicator of A. Then the chemical compounds H_2O, CH_3, CO, CO_2 are singletons in the subsets {H, H, O}, {C, H, H, H} {C, O} and {C, O, O} of this set, as follows.

DEFINITION 5. The set P(**A**) of all singletons σ(**A**) we call *potentially partitive set* of **A**.

EXAMPLE 8. The potentially partitive set of the set **A** = {2, 2, 3, 5} in the example 5 would be:

P(A) = {Ø, {2}, {2}, {3}, {2, 2}, {2, 3}, A_1, $\sigma^1(2)$, $\sigma^2(3)$, $\sigma^3(2, 2)$, $\sigma^4(2, 3)$, $\sigma^5(A)$, ...}.

EXAMPLE 9. Such a set for the set A in the example 6 is:

P(A) = {Ø, {C}, {H} {H}, ..., {O}, ..., $\sigma^1(C)$, A^1, $\sigma^2(H, H, O)$, ..., $\sigma^3(O, O)$, ...}.

Let A be a given set and P(**A**) potentially partitive set of this set. Bearing in mind that the elements of A are sets, it is easy to show that each of conditions in the Definition 5 is fulfilled, so that it could be said for P(**A**) to be a category.

It would be so in the case **A** to be a given fuzzy set.

2. *BIO* [BEING] AND THE CATEGORY THEORY IN MATHEMATICS

Those highest demands, in the spirit of neo-Aristotelism, to be rethought the most diverse creations occurring in the nature, can be followed in an analogous manner within the category theory in mathematics. Thereby the principles of an intuitive area would be confirmed in this essentially different domain, as a formal one, so that – because of its extreme generality – it should be boldly asked: to what extent this mathematical theory can be used as (very) model of Biocosmology? Our answer is that to a high and a highest degree the actions in each of spheres are recognizable in the other one – if now it would be an extreme demand for the very proof of identity (homeomorphisms) of two structures.

Because it is possible to reason that each of the three elements: "Bio", "3" and "4", from the formula "Bio-3/4", as the farthest scheme of Biocosmological researches, finds in the following way its counterpart in the case of category theory in mathematics.

First the prefix "Bio" is the most general expression for the world of living beings at all, as the term "category" is the highest both in logic and in philosophy. This time it passes to mathematics, because several definitions in the category theory appeared to be "powerful" enough to bring out in their entirety dozens of existing theories in it. Namely, not otherwise than a prime example of it (even) occur branches of mathematics such as set theory, group theory (rings, fields, ...), topology, vector spaces, Boolean algebra, deductive system ...

We find here that for the goal we have in mind, to be useful to incline rather to the term of "fuzzy set", than to the notion of set of Cantor, since in the former case it would be respected, with a simple appurtenance of an element to a set, the most diverse properties (powers, values) too, which belong to it, as to an element in the multitude. Herein two corrections of the classical definition of this concept were

made, in so far it allows the existence of the **same** elements here − because nature knows clones (viruses, molecules, DNA copies), and compounds as a special type of union of elements.

Hereupon any whole in the nature consists of parts, and here parts are elements of "partitive sets" in mathematics. Therefore we find it would be most appropriate, to interpret the processes of generations and corruptions in the nature within the category of *potentially partitive fuzzy sets*, which we have outlined above.

3. ONTOLOGICAL ASPECT

3. 1. *On Generation and Corruption*

In the nature we recognize two (essentially) different ways in which elements-parts enter the compositions, make structures and, in general, create a whole. These are:

a) *conjunction* (joining, assembly, association, ...) and
b) *fusion* (permeation, infiltration) [Chapouthier, 2009], that are in the basis of "generation", and two ways in which they come out of them:
a') *disjunction* (separation, disassembly, ...) and
b') *melting* (dissolution, ...) − which are referred to as "corruption".

The first case is illustrated by the examples as:

1^0 pebble stones of different colors which, as properly "joined" in the plane, realize particular figure in a mosaic;

2^0 (each) machine which is it when the parts making it are "assembled" and put in a special (just mechanical) relationship and so on, and the other case the examples as:

3^0 two atoms of hydrogen and one atom of oxygen – which under certain conditions – form a molecule of water;

4^0 blue and yellow color which when mixed together make green color.

*
* *

What happens in the being when the matter is of generation of something new, such are the procedures of conjunction and of fusion of elements as parts into the whole, finds its analogue in the category of potentially partitive fuzzy sets P(**A**), because set operation P of "partitive set" of **A** should correspond here to the operation "conjunction" and to the operation "fusion" − our operation "singleton". Sets know in fact a simple (linear) sequence of elements in the series, as is the case with the partitive set, but it is always possible, by particular functions, some nonlinear arrangement of elements to be translated into linear one. According to the example of a schema of elements in the plane (matrix), which, it can be showed, allows the linear arrangement of them too – by following some agreement – and which could be generalized to each other (more complex) case of such nonlinear arrangements.

So we have that in the case of conjunction, or of fusionn of parts into the wholes, on the way of creation of something new in the nature, it corresponds to writing elements in set theory, so that, both times, the matter is of analogous

structures. But if the matter is of permeating, or of fusion of elements−parts on the same way of generation in the nature, our mosaic understanding of creations, on the one hand, and the possibility to represent them in the way of fuzzy sets, on the other, tell us too that on both sides the matter is of a high degree of analogy between them. For many basic operations (union, intersection, sum, ...) between fuzzy sets, and especially the complex relations between them, make possible to be properly presented the set−result of these operations, which all would correspond to the real relation of parts within a whole.

It would be the case as well when, among several possibilities, one should opt for one of them, as when to some wholeness it should be assigned a certain label (value, probability, truth, ...) too, except that, in contrast, it should be used here the language of so called "fuzzy logic" (**3. 1**).

This time the terms of category of potentially partitive fuzzy sets are intended to express the generation and corruption not only of material but also of spiritual reality of man, for the elements, subsets, sets, on one side, are almost synonymous with elements, parts and wholes, on the other side, as set operations find their counterpart in the conjunction and separation processes in the nature too. For what is new is created by the dismantling of some whole, but when two or more of them "bring" in a particular relation their parts as well. And if we have in mind, especially, finite sets, each choice of elements within them can be made as a result of certain operations on them and to do it − in any case – as an element of partitive set of their union. Insofar as the operations of composition and fusion of elements (decomposition and dissolving), that occur in the nature, can also be expressed in terms of sets, so that to each particular generation (corruption) of something, would corresponded some set relation. The elements, therefore, achieve the wholes and they do it with other elements−entities and so on, what can be schematically presented in Fig. 1.

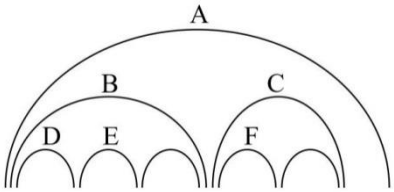

Figure 1

Set relations also can easily be translated into the language of nodes and branches in graph theory, what among others does more attractive the way of "ascending" from parts to the whole and *vice versa*. So the *graph* (see Fig. 2):

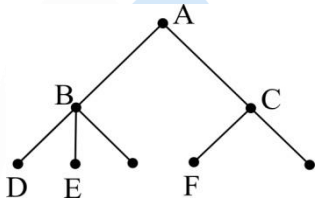

Figure 2

would correspond to the Figure 1, as in fact a set of points and segments in the plane connecting them (or not doing it). The dots are the *nodes*, and its branches (*edges*), as it can be *oriented* if we lay down vectors on its edges.

Then, if we designate by A, B, C, ... edges of an oriented graph, to operation intersection between sets it would correspond here the "transition" from one branch to another, and to the operation union − the transition from a branch into branch, at the same node. Thus to the formulas A ∩ B, A ∪ B and A ∩ (B ∪ C) would correspond the graphs in the Figures 3, 4 and 5, respectively.

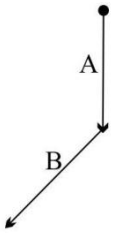

Figure 3

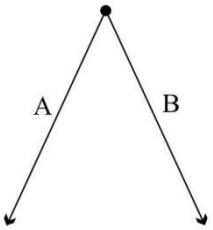

Figure 4

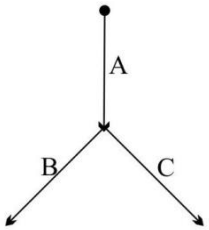

Figure 5

As to the more complex formulas, certainly, more complex graphs should correspond and so on.

As it is known, we can attribute to an edge a *weight* and in a fuzzy set it is corresponded to the *power* of a formula − what all is in the basis of a possible "calculus" over the elements-parts, when they have to realize some particular whole (of a required) power.

(All) nodes and (all) edges of the graph can be brought in a (not) membership relation and to do so a "neighborhood matrix" or a "membership matrix" – in the first case, by assigning the number 1 in the field when a branch connects two nodes, and the number 0, otherwise and, in other case, by attributing the number r ($0 < r \leq 1$), when a node belongs to a branch, and 0 otherwise.

EXAMPLE 10. In the case of the graph in Example 3, represented by a formula A ∩ (B ∪ C), let us denote the nodes by numbers (Fig. 6):

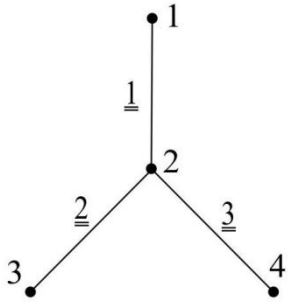

Figure 6

1, 2, 3 and 4, and the edges with bold digits **1**, **2** and **3**. The two aforementioned matrices of elements would be (Fig. 7):

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 3/4 \\ 0 & 2/3 & 0 \\ 0 & 0 & 4/5 \end{bmatrix}$$

Figure 7

3. 2. *One and Many*

We express ourselves most appropriately about the being by saying that it is "one and many", in each of different aspects it has. It was recognized too by the first of philosophers (Thales) when he says: *Hen kai pan* ("One and All") [Dils, Kranz 1974, 11 B 15] so that soon after it Pythagoras's words about would be: "Everything is made from the monad and dyad unlimited" (Dils, Kranz 1974, 58 B 7) and so on, which was a guiding idea throughout history until today, such "bricks of the universe" to be required and found until today. From water of Thales and fire of Heraclitus, through the idea of Plato, until the monad of Leibniz, or the absolute mind in Hegel, that is, elementary particles in physics – or particle which to everyone else gives its mass (boson of Higgs), in the science of physics, which just is seeking for.

Next yet the Eleates set out in epistemology that "true is what is identical to itself", and what took Plato afterwards, saying that if does not allow "an idea always identical to itself, one would not have where to direct his thought" (Gallop 1984, 135 b – c). No knowledge, therefore, would be at all with the variability of all without end, nor would have place giving names to things and beings, for one and the same word would be referred to multiplicity, to multitude. And that "one", that embodies "many" as well, generated the famous "problem of universal's" in the Middle Ages, when one wanted to bend unequivocally to one of its "ontological status", so that, when following Socrates testament, all sciences tend to define terms with which they basically proceed, it would be matter of the same.

More convincing evidence of it come to us from the sciences of physics, chemistry, biology, ... because to our understanding of the nature it belongs things to be made of atoms and molecules – and molecules of atoms − and the living beings of cells, which constitute the tissues and so on, as are discernible too the forces that penetrate it, or fields that populate it.

And in the same way is spoken about the structure of the universe, as it is described so geographical picture of the world it provides.

*

* *

About the category of potentially-partitive fuzzy sets can be spoken in terms of "one and many" as well as on the being, because it brings a whole too through parts that make it up. It is constructed afterwards on the basis of a small number of elements in the experience, according to what they are supposed to provide and what

is potentially contained in them, so that here is recognized that "one", which is in the same time "many" – and *vice versa*. Because if this table is a special whole, built after someone's idea of parts, the same could be said about it when we follow our own taste too − in the looks it has, the function that performs – in the constitution of it. Namely, the taste elements are also parts that take part in such an idea (representation) of the object, within a particular experience. It occurs too when, on the basis of some criteria, we make choices, or when following convictions we judge about something and so on.

3. 3. *Functionality*

What we call functionality of being? The feature of it is that itself, or parts making it repeat the same (or related) structure, so that it is said about them to be analogous, homologous, similar etc., what justifies the expectation, say, the properties of some of them to exist in other parts. For Pseudo−Dionysius: the analogy is a "degree of participation in God's perfection", and for Thomas Aquinas: a way of understanding infinite commencing with finite. Otherwise, Aristotle held that all belong to one of the existing ten categories, and in the *Nicomachean Ethics* kinds of goods such as: measure, pleasant, habitat, ... he deduce from the categories of being: quantity, quality, location, ... At the same time beings (and facts) know the causes and consequences, species and genera, orders of complexity, and (the same) means of emergence and disappearance of something, so that in the latter case it is spoken about: conjunction, separation, permeation etc. In particular, logical categories are characterized by different relations of general and particular, of order and mutual succession, so that all these "identical" aspects of being, at different places (*topoi*), indicate to its uniqueness − to that One which is expressed as Many.

*

* *

This property of being that is in the similarity (analogy) of structures is, first, in that they allow to be represented in terms of set operations, insofar as fuzzy sets make visible "degrees" of properties that parts bring into a whole. What is sufficiently universal feature, whenever the matter is of generation of something new in the nature, and this time we have too that (all) objects of categories we represent by points, and (all) morphisms by arrows. What is a unifying characteristic as well.

Or that parts of being are mutually related as genera and species, here is represented in a manner of an object with more arrows (node with more edges, set with more subsets), so in all cases of genera and species in the same way, what does possible to transfer results obtained in one of cases to other ones of the same gender.

Then to an indirect transfer of the activity from an object to another in the nature corresponds in the theory of category the associativity of morphisms (ordered by definition), as to any operation within the closed universe of objects would correspond the operation of identity in this mathematical theory.

Not only it: the method applied in one theory may prove to be suitable to provide results in another one, or to serve to construct some general (common) theory – as, say, category theory "covered" set theory, topological spaces, vector spaces and

so on. Not to mention the various applications of mathematics in all areas of human activity, or of informatics as being is in the basis of technological civilization today.

3. 4. *Duality*

Of equal importance is the fact that the word "one" and "many", by contradicting each other, give away to a double picture of everything, as an essential duality of the world being, "speaking" out of its depth. The one that in the myth of the origin of the world, in the dialogues *Timaeus* and *Sophist* of Plato, obtains the names of "one" and "other" (different), so that, in the case of other of them, it equally may denote being and non-being, as during the Big Bang, for example, it gives birth to the matter and to the antimatter. There are numerous examples of mutually reducible (dual) concepts, and with one and many we have mentioned – these are: the wave and particle, electric and magnetic field, gravitational and electrostatic forces in physics, point and straight line, side and angle of triangle in mathematics, connectives and or, *id est* for each and some in logic and so on. As it is known, the light gives away by its dual nature: of wave and particle, while the electrostatic law of attraction (rejection) of two electric charges (Coulomb) has the same expression as the law of gravity on attraction of masses (Newton).[1] And to say: "Through two points passes a straight line, or dual to it; or "Two lines intersect at one point", − both times is true. Similarly, if a polygon has three sides, it (necessarily) has three angles and *vice versa* etc., and so we have that dual terms can mutually replace their places and terms containing them remain valid.

This Janus face of universal, therefore, doesn't not give away the one and only one logic – in micro, as in macro world. Classical logic is valid only in a part of reality – so that the whole being is describable rather in terms of topological, functional, local as something most general and primordial. Insofar as the complexity of micro world is such that it allows only the description under the assumption of more than ten dimensions, as is the case with the macrocosm too (fractals), that gives reason to assume that they are governed by different (and so far undiscovered) logics.

*

* *

Category theory, as viewed from several aspects, is a dual system. We have, first, that its axioms still meet if the objects (points, nodes) and morphisms (arrows, edges) mutually replace their places. It occurs if the (actual) direction of arrow is replaced by a reverse direction, and if the transition from edge into edge, by arrow, is attributed by the operation, say, intersection and to the transition of the edge into edge, by node, the operation union, the expression corresponding to the graph remains to be true even when signs of union and intersection mutually replace their places. Therefore, the operation of transition from node into edge and *vice versa*, and of union and intersection are mutually dual to each other, which would be the case with quantifiers "for all" and "there is" as well.

[1] Id est: $F = \lambda(q_1q_2)/r^2$ and $F = \mu(m_1m_2)/r^2$.

EXAMPLE 11. If we alter the direction of arrows on the graph in Figure 8, we get (again) a graph in Figure 9. Or, if the edges and the nodes mutually replace their places, in the graph on figure 6, we obtain a graph in Figure 8, dual to the given one. Here, if the expression A ∩ (B ∪ C) is referred to the first graph, the same expression would correspond to the other one, and to the third graph the expression (B ∪ C) ∩ A. As everywhere too a sign for union could be replaced by the sign for intersection and so on.

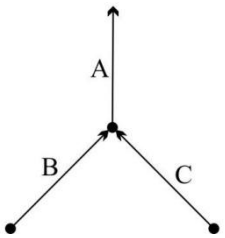

Figure 8

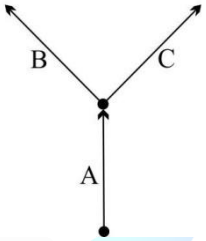

Figure 9

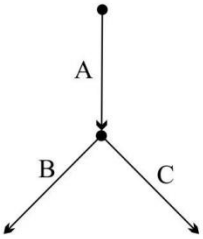

Figure 10

3. 5. *The topological properties*

In what are then the topological properties of things and beings? In that they are essentially determined by other things and beings from their environment, and a result in Mathematics (Diaconescu) shows that the structure itself of the elements of a set determines the logic in it – so "topological" is superposed over "logical". Habitat, environmental conditions determine otherwise animate and inanimate beings in nature (Darwin), as man's generic essence is realized only in a community of people, and words acquire meaning within the context. At the same time Leibniz, Hume argue that *vérités de faits* are not necessary, for "their opposite is possible too", as determined by environment, by place (*locus, topos*) − unlike the *vérités de raison*. As the general theory of relativity confirmed that time is local and dependent of the speed with which the body moves.

So things and beings, bring their essence, but they are better represented by different relationships they "knit" with other things and beings within the environment they belong to.

*

* *

As it is known, the topological space is an example of a category in mathematics, and topology is a developed mathematical discipline, created *via* resolution of a spatial problem (bridges of Königsberg). What it is occupied with are properties of spatial objects, which do not change by continuous changing of forms (like a stretch − but not tearing or gluing).

3. 6. *Emptiness*

Nature knows absence, a void in the micro, as in the macro world, as places which remain when such creations disappear. This is the case with any object at all,

but with conditions and processes in the nature too, since they appear, arise and disappear.

Like a certain energy state (say, state of potential energy), which can give way to another state (state of kinetic energy), since the energy is not lost, but only moves from one form to another. And the same can be said about the forces in nature, which can also be of different degree (potency) – so surely to be of a value zero too. Or even: two charges of the same quantity, but of opposite signs, if they come into a contact will be annulled, as in the case of electrons and positrons, matter and antimatter.

Let us mention as well those philosophic-religious sources recording the beginning of the world in Nothing: Hegel, the Old Testament, or Democritus who, in spite of his notion of atoms permits the existence of empty space and so on.

*
* *

As it is known, Cantor's theory of sets allows the concept of the empty set – as nothing – though it can be said that the physical nothing is a place for creation the materiality, and that in the *macro*, as in the *micro* world, the emptiness is differently understood.

4. GNOSEOLOGICO−LOGICAL ASPECTS

4. 1. *Knowledge, opinion*

When something is said about something, we say on the statements to be "true" or "false", but yet historically Democritus and Plato sharply differed a *true knowledge*, from *opinion*. In this fragment Democritus says: "By opinion color, by opinion sweet, by opinion bitter, but in fact atoms and void" (Diels, Kranz B 125). In turn, the sensual world of Plato is "reached" by opinion (*doxa*), and intellectual world in the way of knowledge (*episteme*). At the same time, Aristotle offers a series of intermediate degrees on the cognitive path, such as: representation (*fantasia*), rethought representation, rational presentation, the presentation−understanding (*hypolepsis*) and not only it.

And if to truth corresponds number 1 and to false number 0, each of the numbers−fractions between them could "carry" here some intermediate degree of truthfulness on.

It would concern the understanding (of degree) in which each of properties (do not) belong to a particular object, or being − because, say, a mountain for us is when it is covered by forest, but when it is not the case as well. And it would not be otherwise neither in attributing values to something that is in the evaluation in the sphere of taste etc.

*
* *

If the class of objects X are propositions, they can also be determined in terms of different degrees of truthfulness on a scale from 0 (false) to 1 (true reliable). Then, to every complex proposition a value should equally belong to the same set, as well as to the rule *Modus ponens* (MP) − as a way in which from propositions of certain

degree of truthfulness one proceeds (concludes) to the truthfulness of another of them. And here are the elements for the sketch of so called "fuzzy logic".

Namely, let p and q be two objects of the class X, with the truth value $\tau(p)$ and $\tau(q)$ from the set [0, 1]. Three new functions *negation* $(\sim p)$, *disjunction* $(p \vee q)$ and *conjunction* $(p \wedge q)$ would be defined as follows:

$$\tau(\sim p) = 1 - \tau(p),$$
$$\tau(p \vee q) = \max[\tau(p), \tau(q)],$$
$$\tau(p \wedge q) = \min[\tau(p), \tau(q)],$$

while in contrast to the classical calculus of propositions, the implications should have the value $\tau(p \rightarrow q) = \max\{1 - \tau(p), \min[\tau(p), \tau(q)]\}$.

In this, a possible rule of inference MP would be:

$$\frac{\tau(p), \tau(p \rightarrow q)}{\tau(q)}$$

etc.

EXAMPLE 12. Let p be the proposition: "Man is a rational being," and q proposition: "Man is a living being." Then on the scale from 0 (not true) and 1 (true), to the first of them will belong, say, the value $\tau(p) = 0.7$ − as the man personifies (more) reasonable, than irrational being – and to the second one the value $\tau(q) = 1$, for he is certainly a living being.

In that case, the value of $\tau(\sim p)$ would be $\tau(\sim p) = 1 - \tau(p) = 1 - 0.7 = 0.3$ and would concern to the unreasonable "part" in man, while the conjunction of these two propositions $\tau(p \wedge q)$ would have a value $\min(0.7, 1) = 0.7$, the disjunction $\tau(p \vee q)$ the value $\max(0.7, 1) = 1$, and the implications $\tau(p \rightarrow q) = \max\{1 - 0.7, \min[0.7, 1]\} = \max(0.3, 0.7) = 0.7$.

5. ***BIO*-3**

As to the "three types of realistically independent spheres of activity,[1] or the sign "3" in the formula "Bio-3/4", when matter is constantly in processes of generation and corruption in nature, we recognize the feature of "three-dimensionality" in the case of all created objects we meet in the experience, because each of them is preceded, first, by a need (associated with some idea), by creation itself afterwards and, ultimately, by its use. Similarly three moments would denote the disappearance of an object too: the loss of the need for it, its destruction and the benefit of that. In the sense attributed to them by Pitirim A. Sorokin we recognize − at this point too − the first and the third moment as "(polar) opposite": the idea and the thing (thing and idea), while the third one is "basic (fundamental)" (Khroutski, 2010): the creation (destruction) itself of things. The scheme, which otherwise, follows every meaningful human activity and is expressed in terms of idea, and (of way) to realize this idea.

[1] Essential propositions of the Biocosmological development: http://en.biocosmology.ru

In our understanding of being as "one and many", two last moments would have the labels of (polar) opposites, while the ways themselves in which elements enter the relations of composition and fusion in creating something new, would belong to a third, autonomous sphere. It can be said about atoms, molecules, compounds, which are in the same time part and whole, as the beginning and the end of a process in the nature, as well as, say, about a family as the fundamental unit of a society, on one side and organized community of people (state), on the other. Or about few propositions (axioms) in the basis of deductive systems, which suffice to generate the wholeness of true statements in them etc., as well as that one and many (parts and whole) preserve their autonomy when they enter compounds and gain a new functionality. In the latter case (many) parts of the being give away a mutual analogy in regard to the internal structure of elements making them, of characteristics which belong to them and so on, so that the aforementioned triad, as an ontological feature of generation of something in a spiritual reality would be in so far reliable way to stretch on the whole of the entire being in general.

And it would be easier to speak about trinity in the processes which are dual to the given one, when recognized in the latter, because both processes have identical structure − only some "key" terms in them would mutually replace their places. ~~For~~ There is a simple symmetry, for example, in the meanings of the words "one" and "many", so all could be said using one of them, – it would be equally valid when it is replaced by the second word.

Finally, and perhaps at the very beginning, we have that already Porphyry has found that all being is experiencing on three sides: in reality, in thought, and in the field of language. In this way, on three sides, the answer was sought and found to the famous "problem of universals" in the Middle Ages. Namely, various philosophers differently considered through history the so-called "ontological status of general notions", and three types of responses one (mostly) meet, had the labels of realism, ~~of~~ nominalism and conceptualism, with regard to these areas, as follows.

*

* *

Next the mathematics brings an expressive picture of bipolarities of idealities it acts with and what is "between" them − which is one tripartism – so that it is even a part of a colloquial expressions about it: "All mathematics is a theory of mapping". Each mapping "includes" two sets − one initial and one final − and a procedure of association of elements of a set (domain) to the elements of the second one (codomain). We have seen these "objects" and "morfisms" in the category theory.

The procedure itself of association is the most diverse and is, rather, an abstract entity, such as, by their nature, the objects being "mapped" into one another are. The domain and codomain can, however, be objects whose nature is the same, as in the case of arithmetic, analysis, because their numbers are here both times. Besides ~~it~~ this, it should be mentioned, its objects appear in different branches of mathematics, including expressions, matrices, vectors, figures, etc., and even the functions or functions of functions (*functionals*), and so on. For example, to a circle of radius *r*, as a line in the plane, on the one hand, we assign the number $2r\pi$, which expresses its

perimeter, on the other. Or even: to a set of *n* linear equations we attribute a "scheme" of elements composed of coefficients of equation, called the "matrix", which, in turn, is not a number, etc.

In mathematics we have also to transform some expressions into the other, when we solve the equation, or, as in the deductive system, certain formulas are translated into the other, for which we say to be true. What we call proof, also is a special transformation, in a permissible way, of expressions to those contained in the formulation of the theorem and so on, so that in each case we recognize just *three* (disparate) moments in this science, in the widest sense: that from which one starts, the way how to proceed with it and to what one arrives in the end.

6. *BIO*-4

In accordance with our intention, a particular mathematical theory to serves as a prevailing model for the process of generation in the nature. Likewise, in the human spirit, let us consider somewhat as the extent to which these processes can be considered through the prism of the four Aristotelian causes, and they are: material, formal, effective and final.

First, in inanimate nature, we have the expression of "blind" activity (in terms of the antique) of proto elements: earth, water, fire and air, or (in the modern interpretation) of the four natural forces underlying the universe: gravity, electromagnetic force, strong nuclear and weak nuclear forces. So far, the equations of quantum theory describes better the reality of micro and macro world than classical equations, and it seems that the future of the world, if at all, can be better described in terms of the theory. What is equally certain on that side would be that a human – in free activity − acquires (unprecedented) power solely to decide about the fate of the planet. Therefore, we inevitably arrive at the question: Has this world a final cause and entelechy? The answer anyway would be that the power of man to realize a free action lies in the (non)reasonable sphere.

Because when the "world is the one" (Tales), and the human is the only rational being in it, no matter how slow the process of evolution realizes itself. A matter is to believe that nature (continues to) try everywhere to create human being who thinks, being in the possession of consciousness, of intellect.

It is easy to recognize all the four (mentioned above) Aristotelian causes. The reason is that exclusively vegetative and animal species in the nature (but not human) can be formulated in terms of the theory of evolution. Namely, every plant and animal naturally has as an objective to survive and to reproduce itself. They are developed on the basis of their genetic codes, under the influence of the environment, through the overlapping of different factors in the given place, of this or that particular power, etc.

In the sphere of activity, again, when the matter relates to objects that are used in experience, and which are created in accordance with some objective they have to serve, which in turn is preceded by the concept (idea) of the thing itself at the beginning and so on. For example: this table of wood (*material cause*), of oval shape (*formal cause*), is made in an industrial way (*efficient cause*) and is used in the dining

room (*final cause*). Or if it is spoken about a purposeful human activity as a (complex) process where it equally would be the case.

*

* *

Finally, we recognize here also the characteristics of quadruple causality of mathematical structures, in the Aristotelian sense of distinction of material, formal and efficient causes of everything. Of these, the latter is in that, say, if it is built a theory, it has to serve to resolve various problems, expressed in its terms.

Or in the case of mathematical object A = {[0, 1], +} and the operation + "addition modulo 2". The material cause here would be the digits 0 and 1, the formal − schema of elements (Fig. 11):

+	0	1
0	0	1
1	0	1

Figure 11

as a special arrangement of them − while the efficient cause would be in the procedure itself of addition of two numbers 0 and 1, and final one − because the operation + does possible the sum of any two numbers, when they are represented in the binary system − i.e. in all that where it can be used.

Thus Aristotle's views on the reality of generation and corruption, in general, are confirmed in these most exact forms of sciences, and it will transfer, no doubt, the same scheme of thinking everywhere where it meets application.

Herein, the language of a set theory is to a high degree recognizable: of elements and compounds (as members of partitive sets), that is, of crossing, of selection and mutation − as operations on sets.

Then, what in the theory of evolution belongs to the "struggle for survival," when an individual more adapted to environmental conditions persist, while others disappear, here it is "followed by" those membership numbers, as special "ponders", "weights", "masses", "degrees", ... allowing it.

7. MAN AND HIS CHOICES IN FREEDOM

Can finally choices made by an individual (in the sphere of freedom) be equally interpreted in terms of category theory in mathematics? It is easy to show it is indeed the case: for there are finitely many opportunities that lie ahead, as (any) choice is only a certain combination of them. And if the first is a set, the second is an element of its partitive set. And partitive sets − as objects − are the categories with respect to the operations of mapping − as morphisms − which would be equally the case when its matter is potentially partitive fuzzy sets.

For the latter "complete" standard definition of set, in so far as they allow the repetition of "same" elements (such as clones, replicators) – say, this table consists of the "same" legs, and two electrons are never in the same quantum state (*Pauli Exclusion Principle*). After, while the first definition of set knows only a sequence of

elements as "isolated" entities, with the notion of singleton function, now any potentially possible entity, composed of one or more of its members, becomes a member of partitive set. Like set A in Example 6, where H_2O, CO, ... as compounds are elements of the set P(A), when C, H and O belong to A.

Furthermore, by performing choices, we adhere to something to which we attach greater importance, since − also subjectively − we assign in thoughts some mark to it from an order of values to each of elements that are offered. In terms of fuzzy sets, it would be their fuzzy numbers, as we are now able to "calculate" fuzzy numbers too of different combinations, as possibilities, realized in the manner of set, or algebraic operations on them. Or we could follow different procedures by way of algorithms, which could bring us to "optimal" forms, of all possible solutions. Since formulas, set expressions, graphs, matrices, ... allow an exact treatment, so, for example, the problem: "Make the best choice of one of four objects with three properties (evaluated from 0 to 1), according to a criterion" could be resolved by using algorithms called Elektra, Prometheus or AHP. It would correspond to a matrix with four rows and three columns in the Example 6 etc., so that a singular case of the problem would be, say: "Decide purchasing one of four types of printers, taking into account: the number of nozzles, pricing and quality print". Finally, on the same way, the processes of transition from one or more possibilities to a new one, being "graded" − as more or less likely – are those in fuzzy logic and that we have outlined it in Section 4.1.

In terms of Aristotelian fourfold reason of everything existing, such a choice , as a subset, is the sign of final cause, while material cause is provided by opportunities, formal – by the very nature of the product, and efficient – by an algorithm carrying us to it. It is also tripartite one: here the initial elements of the choice are different; the manner in which it is realized, as well as the way in which we arrive to it, afterwards.

References

Baez, John C. (1997). *An Introduction to n-Categories*. Riverside: Department of Mathematics, University of California.

Chapouthier, Georges. (2009). Mosaic structures – a working hypothesis for the complexity of living organisms. *E – logos. Electronic Journal for Philosophy*. Prague: University of Economics. URL: http://e-logos.vse.cz/ (retrieved May 12, 2012)

Diels, H. and W. Kranz. (1974). *Die Fragmente der Vorsokratiker*, three vols., original edn. 1903; reprint of 6th edn., Berlin: Weidmann.

Khroutski, Konstantin S. (2010). On Biocosmology, Aristotelism and the prospects of becoming of the universal science and philosophy // Electronic journal "*Biocosmology – neo-Aristotelism*". Vol. 1. No1. Winter 2010: 4–17. URL: http://en.biocosmology.ru/electronic-journal-biocosmology---neo-aristotelism (retrieved May 12, 2012)

Essential propositions of the Biocosmological development. URL:

http://en.biocosmology.ru/ (retrieved May 12, 2012)
Gallop, D. (1984). *Parmenides of Elea: Fragments*. Toronto: University of Toronto Press.
Zadeh, Lotfi A. (1965). *Fuzzy sets*. Information and Control, 8 (3): 338–353. Atlanta: Elsevier Inc.